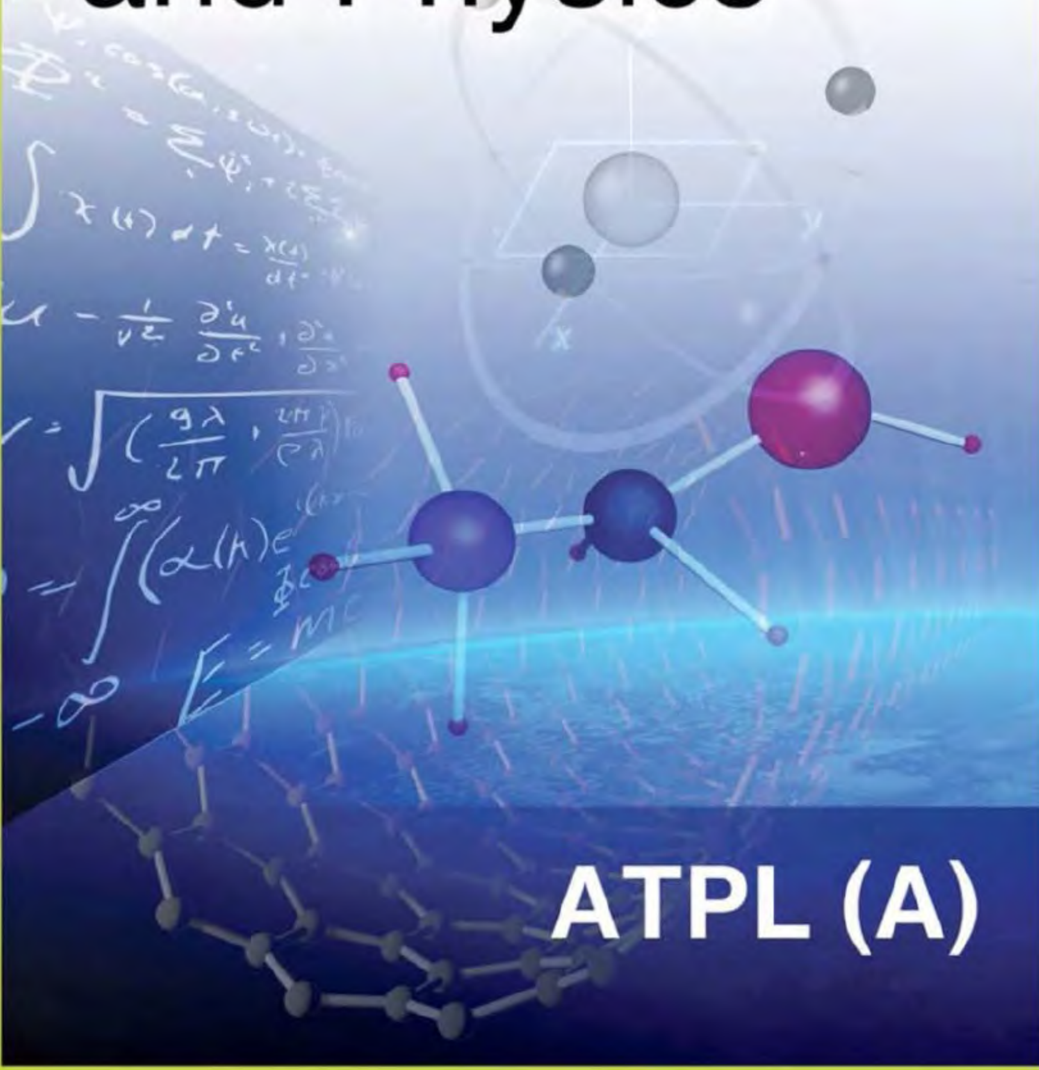
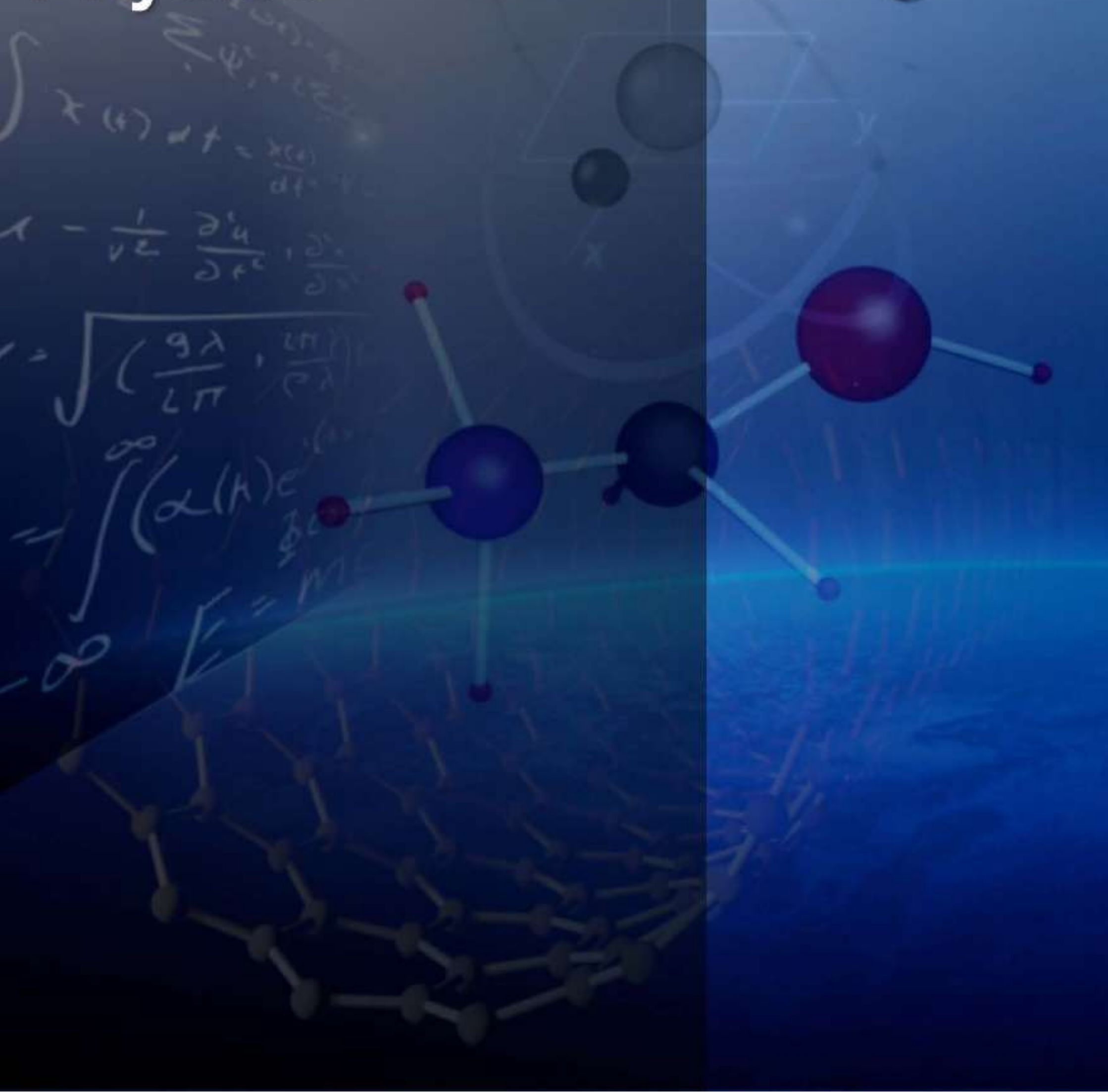
Maths
and Physics



ATPL (A)

Maths and Physics



Foreword

Thank you for downloading this book. We hope it will help you to develop a solid foundation for your PPL or ATPL studies.

Some basic understanding of physics is essential for all pilots, whether they are flying professionally or just for pleasure. Similarly, every pilot needs to have a basic ability to do mental calculations and perform simple mathematical operations.

This book is written by pilots for pilots and is designed to give you a basic grounding in all the essential mathematical and physical principles.

The subjects covered in this book are a mandatory part of the EASA ATPL(A) syllabus and, as such, form an integral part of our PadPilot ATPL(A) course.

But this book is equally important as an aid to all students of aviation including those taking their PPL theory exams. If you fully grasp the concepts explained in this book you should have no difficulty with your future studies.

About This Book

Maths and Physics for Pilots

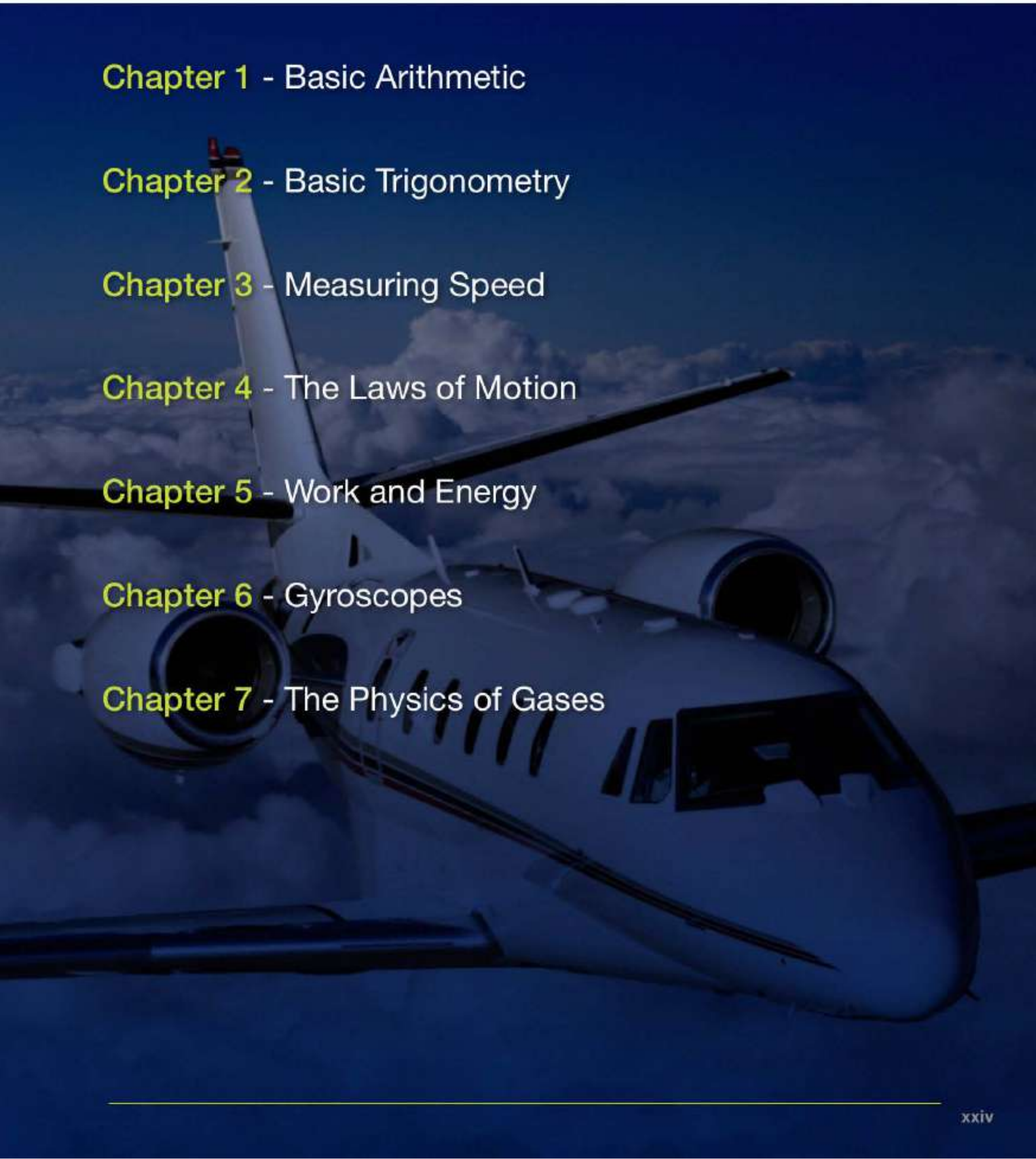
Version 2.2

Published by:

Slate-Ed Ltd
Unit 19, The Glenmore Centre
Gloucester, UK. GL2 2AP

978-1-909600-00-3

Table of Contents



Chapter 1 - Basic Arithmetic

Chapter 2 - Basic Trigonometry

Chapter 3 - Measuring Speed

Chapter 4 - The Laws of Motion

Chapter 5 - Work and Energy

Chapter 6 - Gyroscopes

Chapter 7 - The Physics of Gases

1

Basic Arithmetic

The maths and physics used in aviation is usually covered in school. You will probably find the maths much easier than expected. If you don't, this book is here to help!

This first chapter concentrates on elementary mathematical principles. Some of these are very basic indeed but there's no harm in refreshing your memory about the essentials!

Section 1

Basic Arithmetic

1.1.1 Introduction to Maths and Physics

The maths and physics used in aviation is usually covered in school. However some of you may have found these subjects a little bit daunting. Or perhaps, as the years pass, your knowledge has become rusty with lack of application. The purpose of this very quick course is to revise and practise the areas of maths and physics that you will need for your ATPL course.

You will probably discover that you find the maths much easier than expected, if not help is always at hand for our students! This first chapter concentrates on elementary mathematical principles. Some of these are very basic indeed but there's no harm in refreshing your memory about the essentials!

The next chapter introduces basic trigonometry and the following chapters discuss basic concepts of physics which are applicable to almost all sections of the ATPL syllabus. Like basic maths, understanding these concepts is essential to your studies. If you don't grasp them fully you will not be able to understand the fundamentals of almost all the ATPL subjects. Consequently, if you are at all unsure about your competence in any of these topics please read this book carefully.

1.1.2 Order of Operations

The first thing you need to know about basic arithmetic is that we must do the various operations in a specific order. To understand why we'll look at two possible outcomes from one apparently similar statement:

3 x 4 + 5 = ?

This is ambiguous because it could be asking you, for example, to find out what happens when you add five to 3 x 4. Answer = 17.

But it could also be asking you to find out what happens when you multiply 4 + 5 by three! 9 x 3 is 27.

In fact to a mathematician (i.e not us) the question is completely unambiguous because they know that all calculations are processed in a very specific order, as follows.

Firstly, you must do any calculations which are inside brackets, then do divisions, then multiplications, then addition and finally subtraction.

So, in the above example, which has no brackets, the multiplication is done first to give 12 followed by the addition to give 12 + 5 = 17. You can remember the order of doing things using this simple mnemonic:

BODMAS:

Brackets off, **D**ivide, **M**ultiply, **A**dd, **S**ubtract

To make it absolutely clear what we need to do, we can add brackets. These help you to understand how to process the statement. So, our example with brackets added would look like this:

$(3 \times 4) + 5 = ?$

Now, the problem becomes unambiguous. Answer = 17.

One of the important facts BODMAS tells us is:

> Always do the calculations inside brackets before you do anything else.

1.1.3 The Equals Sign

The next really important thing to understand is the meaning of the equals sign (=).

4 + 4 = 8 is a statement of fact. The equals sign does NOT mean "the answer is...".

This may seem like a very fine distinction but it becomes helpful and important when we move onto our next topic, equations.

The equals sign means that whatever you have put on one side of the sign has exactly the same value as whatever you put on the other side of the sign.

Section 2
Equations

1.2.1 Equations

An equation is a method of showing that the outcome of two things is the same, or equal. Quite literally it means that one side of the equals sign *equates* to the other.

For example look at this simple equation: 10 = 5 x 2

You can read it from left to right, or from right to left, the outcome is the same: 5 x 2 is the same as ten, or ten is the same as 5 x 2.

You have to be a little bit careful about how you read the equation. Look at this equation for example:

10 = 12 - 2

This can be read as "12 - 2 is the same as ten" or "ten is the same as 12 - 2." But 2 - 12 is not the same as ten!

So the first rule about equations is that you must used BODMAS to deal with the arithmetic on each side of the equals sign.

For example

10 - 2 = 4 + 2 + 2 made simple this can be read as:

8 = 4 + 2 + 2 or 4 + 2 + 2 = 8

Or, of course: 8 = 8!

1.2.2 Substituting Numbers for Letters

Sometimes we can't write equations using numbers ("values") because we don't know some, or all, of the values. In this case we can substitute a letter for each value that is unknown. It can be any letter you like - except that in our ATPL studies, by convention, specific letters are used to represent specific things.

For example the Greek letter theta (θ) is often used to represent the value of an angle. There's nothing at all special about theta - it's just that Principles of Flight teachers conventionally use this symbol instead of some other letter.

Letters are also used when we never want to quantify the result of an equation. Sometimes all we want to do is understand the relationship between various factors. For example look at this equation

$T = D + p$

This equation shows the relationship of forces along the flight path in a climb. In this case T stands for Thrust, D for Drag and p for a proportion of the weight.

We will never want to know the values for each of these components because we aren't aerodynamic engineers. Nevertheless even with letters instead of values, the equation usefully tells us in broad terms that the amount thrust which needs to be produced in a climb is related to the drag produced by the aircraft and the weight of the aircraft.

1.2.3 Substituting Letters for Numbers

On other occasions we very much need to know the values in an equation. For example, we need to load 50 English gallons of fuel into our aircraft but the fuel bowser is calibrated in litres. How many litres of fuel do we need to load?

We first need one fact: that there are 4.55 litres in a gallon.

Now we can construct a very simple formula using L for litres and G for gallons.

$L = G \times 4.55$

Now all we have to is substitute the "G" for our known value of 50 and then do the arithmetic. So:

$L = 50 \times 4.55$

$L = 227.5$

We need to load 227.5 litres of fuel from the bowser.

1.2.4 Simplifying Equations

In the previous example we had an easy equation to deal with. We wanted to find out the number of litres to load and the equation could tell us that directly because it started with the expression "l =".

Unfortunately life isn't always that simple. Sometimes equations have arithmetic to do on both sides before you can come up with a simpler form that can tell you what you need to know.

1.2.5 Adding and Subtracting

To see how this works in practice, we'll use for our example a simple three element equation.

$T = D + p$

Let's say that we want to use this equation to find the value of D. To say *"find the value of D"* is the same thing as saying that we want to find what D is *equal* to. Which means that we need to rearrange this equation so that it starts as: "D =" (or ends with "= D").

To get D by itself on the right-hand side of the equation, we need to subtract p. Subtracting something from itself always results in zero and letters are no exception; p - p equals zero so it will disappear from the right side leaving D all on its own. This is no different from saying, for example, that +10 added to -10 equals zero.

But if we want to subtract p from the right side of the equation we must also subtract it from the left side. Whatever you do to one side on an equation must always be done to the other. This is an absolutely essential point so it's worth stressing again:

> Both sides of an equation must be equal. If something is done to one side of the equation the same must also be done to the other side

So now our equation looks like this:

$T - p = D + p - p$

The "+ p" and the "– p" on the right-hand side equal 0 so we can cancel them out.

So now we are left with a much simpler looking equation which also does exactly what it needs to do - it tells us that D is equal to something.

$T - p = D$

Another useful way of thinking about what we have just done is to say that whenever an element of an equation changes from one side of the equation to the other, it must change its sign. So in our example "+ p" became "- p" when it changed sides. This idea becomes very useful we start to manipulate equations with multiplications or divisions in them.

1.2.6 Multiplying and Dividing

The opposite sign to multiply is divide. So, if we wish to swap an element which is being multiplied to the other side of the equation we need to change it's sign to divide. Let's look at one of the most famous of all equations, Isaac Newton's:

$F = m \times a$

(Force equals mass times acceleration)

Remember that mathematicians usually don't include the multiply sign. So the formula would, more usually, be written as $F = ma$, or $F = am$. Whenever you see two elements of a formula side-by-side with no arithmetic signs, assume that they should be multiplied together.

If we want to find the value of "a" we need to move the "m" to the other side.

So: $F = m \times a$ becomes $F \div m = a$

You might be wondering how we knew that m was a "x m" and not a "+ m" or "- m". Multiplication may be read both ways so the original formula could actually have read:

$F = a \times m$

Section 3

Thinking in Boxes

1.3.1 Dividing a Formula into Boxes

When we manipulate formulae its often useful to treat each element of the formula as if it's contained in its own little box. Then all you do is move the boxes around, making sure that any sign contained inside is reversed. So:

F	=	A	xM

Can become:

F	÷M	=	A

Or

F	÷A	=	M

1.3.2 Cross Multiplying

For more complicated formulae we are better off using a technique called cross multiplying. Again putting things in boxes helps us to visualise what's going on. We'll also revert back to our original method for manipulating a formula: applying a change equally to both sides. F = m x a. To find "a" we need to cancel out "m" by dividing it by itself:

F	=	m ÷ m	x a

Oops, we must do the same thing to the other side of the equation:

F ÷ m	=	m ÷ m	x a

We'll replace the division signs and use conventional fractions. So:

$\frac{F}{m}$	$\frac{m}{m}$	=	x a

The m/m on the right hand side now cancels itself out

$\frac{F}{m}$	$\frac{\cancel{m}}{\cancel{m}}$	=	x a

Let's look again at what just happened. We rearranged the formula like this:

F	=	m	x a

$\frac{F}{M}$	=	A

This is cross multiplying because "m" changed from the top to the bottom when it moved across the equals sign.

We can apply this cross multiplying idea to a more complicated formula. For example, from this formula we want to find "a".

$\frac{a \times b}{c}$	=	$\frac{d}{e}$

We just move "b" and "c" to the other side using cross multiplication.

$\frac{a \times b}{c}$	=	$\frac{c \times d}{b \times e}$

a	=	$\frac{c \times d}{b \times e}$

Easy!

With our new found skills we are now ready to rearrange a formula which contains multiplication and subtraction. Let's look at this one:

$$\theta = \frac{T-D}{W}$$

Though you don't need to know it, this formula tells us that the angle of climb theta (multiplied by another factor which for clarity we haven't shown here), equals Thrust minus Drag divided by weight.

To find the value of Thrust we need to need to do this in two stages.

Θ	=	$\frac{T - D}{W}$

Firstly, we'll cross multiply W to get a formula that only has one line:

Θ x W	=	T - D

Because we know that we're going to be adding more to this equation it will help if we write this with brackets:

(Θ x W)	=	T - D

Now we can get rid of D by adding it to both sides of the equation and then cancelling it out on the right hand side:

(Θ x W) + D	=	T - ~~D + D~~

(Θ x W) + D	=	T

Changing Elements of an Equation

1.4.1 Changing The Value of Elements of an Equation

The equals sign in an equation carries some other important implications concerning what happens when we change the value of an element in an equation. We need to look at two cases:

- Changing the value on the opposite side of the equation
- Changing one of the values on one side of an equation

Let's look at the simplest possible example of the first case: 5=3+2

If we change this to 6 = 3 + 2 the equation no longer makes sense. The increase in the value on the left side of the equation must result in an increase in value on the right side. For example: 6=3+3

Now let's look at the second case. Changing elements on one side of the equation only. Again let's take a very simple example

10 = 6 + 4

Now what happens when we need to change the value "6" to the value "7" without altering the "10" on the left side of the equation.

10 = 7 + 4 doesn't work, so the "4" must become a "3"

> Changing one side of an equation only means that if the value of one element increases, the value of the other element(s) on the same side of the equation must decrease by the same amount

With examples as simple as the ones above it's easy to see the logic. But most of the time in the ATPL course we aren't dealing with numbers, just letters representing different things. Look at this formula for example:

Total Pressure = Dynamic Pressure + Static Pressure

You may already be familiar with this formula, but you don't have to be to understand its implications. For instance, we can say that if Total Pressure remains constant but Dynamic Pressure increases, then Static Pressure must reduce by the same amount.

Alternatively we could say that if Dynamic Pressure increases and Static Pressure stays the same then the total pressure must increase.

We can illustrate these two cases with arrows:

Total Pressure = Dynamic ↑ Pressure = Static ↓ Pressure

Or

Total ↑ Pressure = Dynamic ↑ Pressure = Static Pressure

Proportionality

1.5.1 Proportionality

When it is necessary only to understand the relationship between factors we can sometimes talk about the relationship as being either *proportional* or *inversely proportional*.

A proportional relationship means that if one factor increases, so too does the other in such a way that one factor is a constant multiple of the other.

For example weight expressed in kg is proportional to weight expressed in lb. One kg is approximately 2.2lb. 10 kg is approximately 22 lb, 40 kg is approximately 88 lb etc.

In this proportional relationship lbs are a constant (2.2) multiple of kgs. In other words a proportional relationship is always a multiplication.

lb = kg x 2.2

When two factors are proportional the relationship is described using the proportional sign: $\propto$.

In our pressure equation above, static pressure is not *proportional* to Total Pressure because it doesn't vary as a multiple.

If Total Pressure = static pressure x dynamic pressure the relationship would have been proportional but it doesn't.

If two factors are inversely proportional, it means that if one factor increases, the other factor decreases.

For example the time it takes you to get somewhere is inversely proportional to your speed. The greater your speed the less time it takes to get to your destination.

These simple rules apply to simple equations where everything is on one line. So what happens when the equation has top and bottom lines? For example:

$$\frac{a}{b} = \frac{c}{d}$$

To find out the nature of the relationship you need to rearrange the equation to a single line:

$a \times d = c \times b$

Now you can see that:

- a and d are both proportional to c and b
- a is inversely proportional to d
- c is inversely proportional to b

Section 6

Square and Square Root

1.6.1 Square and Square Root

The square of a number is its value multiplied by itself. For example the square of 4 ("4 squared") is 16 (4 x 4).

In mathematical formulae the squared number is indicated thus: 4^2.

The *square root* of a number is whatever value when multiplied by itself equals the number. For example the square root of 16 is 4 because 4 x 4 is 16. The square root is indicated with the root sign. For example:

$\sqrt{16} = 4$
$\sqrt{4} = 2$
$\sqrt{9} = 3$

Section 7

Indices

1.7.1 Indices

Indices are used to express very large or very small numbers, in a form of short-hand notation. In the ATPL syllabus we are only concerned with indices based on 10, but the principle applies to any number.

You are probably happy that 2^2 ("2 squared") means 2 x 2 which is 4. Similarly 5^2 ("5 squared") means 5 x 5 which is 25. But what does 2^3 or 5^4 mean?

2^3 means 2 x 2 x 2 which is 8. So we could, if we wanted, represent 8 as "2^3".

In this case the number '2' is the *base* being used and the small superscript number is the *index*. The two together are known as the *exponent*

Similarly 5^4 means 5 x 5 x 5 x 5 which is 625. Again we could represent 625 as "5^4".

In simple cases like this there isn't much point in writing the numbers in this short-hand form. It is easier to write and understand "8" than it is to write "2^3".

The system becomes much more useful when we want to describe larger numbers. For example, in aviation, the probability of an "extremely remote" event happening is one in 10 000 000.

1.7.2 Indices Based on 10

Indices based on 10 ("Base 10") are common in technical aviation material so we need to understand them. As always, we'll start with the simplest example:

10^2 is 10 x 10 which is 100. So we can represent 100 as "10^2".

10^4 means 10 x10 x10 x10 which is 10000.

1 000 000 is represented by 10^6.

You've probably noticed by now that the superscript number ("the index") exactly corresponds to the number of zeros. More precisely, it's telling you where to put the decimal point.

Let's look at 10^6. The "6" tells us that we need to move the decimal point six places to the right.

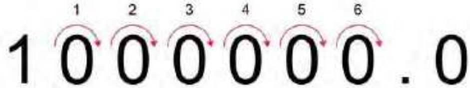


Figure 1.1 A positive index number tells you how many places to move the decimal point to the right

Six places to the right means six places to the right of the "1".

This is reasonably easy when you have simple multiples of ten, like 1 000 000. It not so easy for other numbers. So to make the notation easier to understand, we write 1 000 000 as "1 x 10^6". Which literally means "*write the number 1.0 then move the decimal 6 places to the right*".

Using this notation how might we describe 934 000 000?.

First put the decimal point in its correct place: 934 000 000.00

Now count back until the decimal point comes immediately after the first digit:

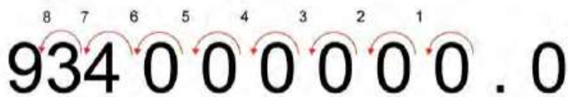


Figure 1.2 Count left until you reach the last number to find the value of the index

We had to move the decimal point eight places to get it to come after the first digit, so 934 000 000 is written as: 9.34×10^8

For example, when using this method 540 000 becomes 5.4×10^5.

Just to get this absolutely clear let's see what 1.25×10^6 means.

Starting with 1.25 000000 move the decimal point six places to the right:

Answer: 125 0000.0 (written in the UK as 1,250,000).

1.7.3 Working with Indices on Calculators

Calculators use indices to Base 10 only. Most calculators will only start to show indices when you are working with very large numbers which exceed the display's available digits. Try entering 885 000 squared onto your calculator now. (The answer is 783 225 000 000).

Depending on what type of calculator you have you will see something similar to the following:

- An expensive graphical display type calculator will probably either display 7.83225×10^{11} or 7.83225^{11} because it assumes you know that the base is always 10.
- A decent calculator, but one which uses an individual cell to display each number will probably display something like 7.83225e+11. In this case the e stands for exponent so the calculator is telling you that the number is 7.83225 times the exponent 10^{11}.
- Alternatively, instead of using the letter "e" you might see one of a variety of other symbols used to denote the exponent, followed by the index number.
- If you're really unlucky your calculator will display something like E 7832.2500. This is telling you that it has recognised that the number needs to be described with an exponent but that it can't do it!! If your calculator is in this category get another one. But remember that programmable calculators are not allowed in the EASA exam.

1.7.4 Using Indices to Represent Very Small Numbers

Indices can also be used to represent very small numbers by showing the power sign as a negative number.

We know that 1×10^2 means 100, but 1×10^{-2} means 0.01

As before, the index tells you how many places to move the decimal point. The minus sign means "divided by" and tells you to move the decimal place to the left not the right. For example:

One millionth is 1 divided by 1 000 000 or $\frac{1}{1000000}$

This can be expressed much more simply as 1×10^{-6}

A calculator display will show 1×10^{-06}, or occasionally $1.^{-06}$.

Now enter 1/250 000 000 in your calculator. You'll probably see something you weren't expecting. Instead of displaying 2.5 x 10 to the something, it will read 4×10^{-09} (or 4e-9 which means the same thing).So what's going on here?

Let's look again at this fraction: $\frac{1}{250000000}$

Before we can express it with indices we need to do some work on it. This is because we can't end up with an expression like $1/25 \times 10^{-7}$ (well, you probably could but mathematicians don't like it!)

Every time you express a number using indices the expression always starts with a whole number. It might have decimal places after it (e.g 4.35) but it *always* starts with a whole number.

The first step then is to ignore the zeros for a moment and just convert the fraction 1/25 into a decimal number. The answer is 0.04.

If we now count the number of zeros, we get our first attempt at an answer: is 0.04×10^{-7}. But we aren't allowed 0.04, we must use 4. This takes two more moves of the decimal place, so the answer is 4×10^{-9}.

Section 8

Symbols

1.8.1 Symbols to Denote Greater Than and Less Than

Sometimes we need a basic symbol that shows that something is greater or less than something else. In this case, because they are not equal, the equal sign cannot be used.

When the left side of an expression is greater (or the right side is smaller) we use the sign ">". For example $7 > 3$.

Similarly, when the right side is greater we use the sign "<".

For example $3 < 7$.

This notation method is commonly used in our course. For example when an aircraft climbs, its thrust is greater than drag: $T > D$.

Section 9

Percentages

1.9.1 Percentages

A percentage is a way of describing the proportion of something, expressed as a fraction of 100.

The word "percentage" stems from the Latin "cent" meaning 100. Whenever you see the word "per" it means "divided by". So "percentage" literally means "divided by 100". And whenever you see the word "of" being used in conjunction with a fraction is means "multiply by".

So, for example, the phrase "50 percent of 60" means:

$\frac{50}{100}$ multiplied by 60

When you have a calculator the simplest way to deal with any percentage problem is to treat it as a decimal.

50/100 is 0.5 so, in the above example, 50% of 60 is 60 x 0.5 which is 30.

Example: what is 20% of 40?

20% = 0.2

0.2 x 40

Answer = *8*

We can use percentages to describe an increase in a quantity.

For example, if your speed is 100kt and you increase it by 10kt to 110kt it has increased by an extra 10/100th - or 10%.

A very useful way of looking at this example is to understand that your new speed is 110% of the old speed.

Example: your current airspeed is 90 kt, you need to increase it by 10%. What will be your new speed?

After the increase your new speed will be 110% of the old speed.

$$\frac{110}{110} \times 90$$

Make this easy for the calculator 110/100 = 1.1 = 1.1 x 90

Answer: *99 kt*

Similarly, if something reduces by 20%, it gets smaller by 20 parts. Therefore, it is now 100 - 20 parts = 80 parts, or 80%, of its original value.

Example: what is 80% of your current speed of 100kt?

$$\frac{80}{100} \times 100$$

Make this easy for the calculator 80/100 = 0.8 = 0.8 x 100

Answer: 80 kt

Percentages are often used to describe the effect of a factor on a distance or gradient (for example the wet landing distance compared to the dry).

Typically, the distance needed to land and stop on a wet runway is 15% greater than on a dry runway. This means that if the original dry landing distance (100%) is 2000 ft, the wet landing distance would be:

$$\frac{115}{100} \times 2000\text{ ft} = 1.15 \times 2000 = \textit{2300 ft}$$

1.9.2 Squaring Small Percentage Changes

Airspeed often has a square effect on other variables, such as take-off distance. For example the take-off distance is proportional to the take-off speed squared.

If the take-off speed increases by 10%, it is now 1.1 times its original speed.

This means that the take-off distance is now $1.1 \times 1.1 = 1.1^2 = 1.21$ times its original length.

The take-off distance is now 21% longer, which is approximately 20% longer.

For small percentages only, the effect of squaring it is very similar to doubling it. For example a 10% increase in speed results in *approximately* a 20% increase in take-off distance. A 5% decrease in speed results in the take-off distance decreasing by *approximately* 10%.

Similarly square rooting a small percentage looks like it has halved. For example stall speed is proportional to the square root of the aeroplane's weight. Therefore if the weight reduces by 10% the stall speed reduces by *approximately* 5%. This is because $\sqrt{0.9} = 0.949$, which is approximately a 5% reduction.

These approximations can be very useful rules of thumb for quick mental calculations.

2

Basic Trigonometry

Triangles are very useful tools in aviation maths because there are very distinct relationships between the sides of a right angled triangle and the angles inside it.

In practical navigation we are often very interested in finding angles, such as the angle of drift or the angle of closure to track.

The Geometry of Triangles

2.1.1 Introduction to Trigonometry

Trigonometry stems from the Greek word for a triangle '*trigonon*'. The word describes study of the properties of triangles.

Triangles are a very useful tool in aviation maths because there are exact relationships between the sides of a right-angled triangle and the angles inside it. In practical navigation for example we are often very interested in finding angles such as the angle of drift or the angle of closure to track. Without some giant protractor in the sky we have no means of measuring these angles; but we can calculate them very accurately provided we have other information about the sides of the triangle. This is why many navigation problems are broken down into a *triangle of velocities*.

In every case, whenever we are talking about triangles in an aviation context we are talking about right-angled triangles. As we shall see in the next chapter it's possible to find a right-angled triangle out of almost anything. We can also find a right-angled triangle in any other triangle. Figure 2.1 illustrates.

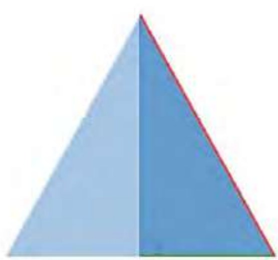


Figure 2.1 Creating right angled triangles

Our first task is to understand the naming conventions for right-angled triangles.

2.1.2 Naming the Sides of a Right-Angled Triangle

The longest side of a triangle is always the side opposite the right angle. This is called the *Hypotenuse* (H). The name stems from the Greek words for 'under' and 'stretch'.

The other two sides of the triangle are known as the *Adjacent* (A) and the *Opposite* (O). Which is which depends on what you want to do.

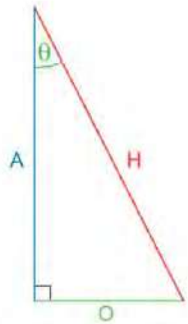


Figure 2.2 The right-angled triangle

If you need to find one of its angles then the next task is to decide which angle it is that you want to know about. In the example above, we want to find the angle at the top of the triangle.

In our example we've named it theta but you might call it alpha (the usual letter for angle of attack). Or you could call it D for drift or S for slope or any other name you like! Once you have decided on the angle of interest, you can now assign the names to the other two sides of the triangle. The side that is opposite to the angle θ is known as the Opposite (O). So by a process of elimination the other one must be the Adjacent (A).

In the next sections we'll see why you need to understand and remember these naming conventions.

2.1.3 Pythagoras

Pythagoras, an ancient Greek mathematician, came up with a startling discovery.

The square of the length of the Hypotenuse always equals the square of the Adjacent plus the square of the Opposite

Expressed as a formula: $H^2 = A^2 + O^2$

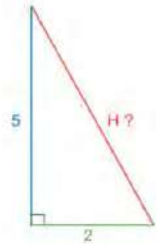


Figure 2.3 Using Pythagoras' theorem to find H

Example: O is 5 and A is 2. What is the length of H?

"O" squared is 25. "A" squared is 4. 25 + 4 is 29. So, the length of the hypotenuse squared is 29. Don't make the easy mistake of forgetting that the result of your calculation is the square of what you actually want. To find the length of the hypotenuse not squared, use the square root function on your calculator. The square root of 29 is 5.385.

The length of the hypotenuse is therefore *5.385*.

This might have been the solution to a navigation problem. You flew for a 5 miles, after which you realised that you were two miles left of track. Overall then, you have flown a total of 5.385 miles.

2.1.4 Similar Triangles

If two triangles have the same internal angles as one another then they will be the same shape. This is true regardless of the size of the triangles. Such triangles are known as *similar triangles*.

Now here's the interesting bit. If two triangles are the same shape, then this also means the relationship between the lengths of their sides will be the same. For example notice that, in the small triangle in Fig 2.4, the Adjacent is 1 unit long and the Hypotenuse is 2 units long. Put simply this triangle has a ratio of 1:2 between its Adjacent and Hypotenuse.

The larger triangle has the exactly the same ratio for the same sides. Although the lengths are now 2 and 4 respectively the ratio is still 1:2. (You could call it 2:4 but by convention we always reduce ratios down to their smallest whole numbers).

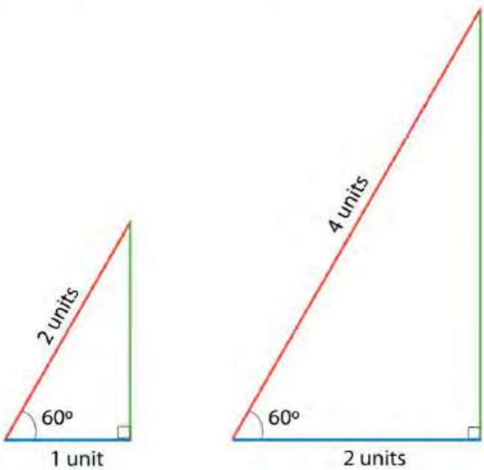


Figure 2.4 Similar Triangles

If you measured the angle you would see that it is 60°.

This is very useful because it means that any time we have a right angle triangle with Adjacent and Hypotenuse in the ratio 1 to 2 we *know* that the angle is 60°.

Alternatively, if we happened to know that the angle was 60° then if we knew the length of either the Adjacent or the Hypotenuse we could work out the length of the other one.

You can think of ratios as fractions. In our example the ratio of the Adjacent to the Hypotenuse can be expressed as:

$$\frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{2}$$

Using our example above, we can say that whenever you have a triangle with an angle of 60° the Adjacent will always be exactly half (0.5) the length of the Hypotenuse.

Or alternatively, if you know that the Adjacent is exactly half the length of the Hypotenuse you can say with absolute certainty that the angle is 60°.

Mathematicians have recorded the relationships for a huge number of angle values related to the adjacent/ hypotenuse and stored them in fixed memory in your calculator. They call it the *Cosine* function ("Cos" for short). Try using it now.

Type in "60" to represent our angle of 60° now press the "Cos" button.

The result is 0.5 - a ratio of 1:2

The cosine function only describes the relationship:

$$\frac{\text{adjacent}}{\text{hypoteneuse}}$$

and not hypotenuse/adjacent or any other combination of the three sides of the triangle. Remembering this, we can safely say that with an angle of 60° the Adjacent is 1/2, or 0.5, of the size of the Hypotenuse.

You can do this for any angle you like using the Cos function of your calculator. Type in "30" then "Cos" and you'll see that the answer comes out at an uncomfortable 0.866. Don't panic. This is simply telling us that the adjacent is 0.866 of the size of the hypotenuse.

On most modern calculators you can do the reverse calculation to find the angle when you know the relationship between the numbers for the Adjacent and the Hypotenuse. The dedicated function button which does this for you is usually nowadays called "Cos-1".

If there isn't a dedicated Cos-1 button then there will be an INV (*Inverse*) button which does the same thing but requires two steps.

For example, if we want to find the angle associated with a relationship of Adjacent/Hypotenuse of 1/2. Depending on your calculator you can do it in one of two ways:

- *Cos-1 button*. Enter 0.5 then press the "Cos-1" button. The answer will be 60. You'll need work out for yourself that 60 means 60°!
- *INV button*. Enter 0.5 then "INV" then "Cos".

If you haven't got a calculator with either Cos or INV then check your user manual. Some scientific calculators use the term "arc-cos" to mean the inverse of cos. If your machine really has no Cos and Cos-1 functions then go and buy one that does!

2.1.5 Practical Aviation Example of Trigonometry

Finding the angle when we know the length of two sides can be very useful for us pilots. Let's take a second look at the example which first appeared in Fig 2.3. We've turned it upside down so that it makes a little more sense as a navigation problem. We've also drawn it very precisely; you can check it with a ruler and protractor!

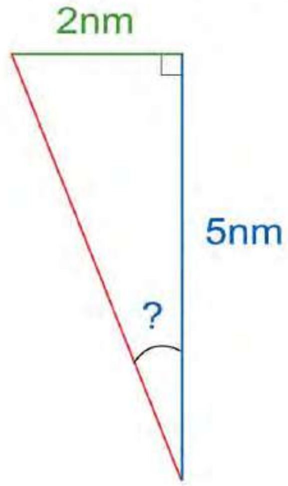


Figure 2.5 Using the Cosine function to find the drift angle

We flew for 5 miles, after which we found out that we were 2 miles left of track. What is the angle of error between our intended track and the path we actually flew?

A problem becomes immediately apparent - we don't know the length of the hypotenuse so we can't use our cosine function.

We could calculate the length of the hypotenuse using Pythagoras but that would be tedious and time consuming. Luckily again for us, mathematicians have already calculated the relationships for all three combinations of sides.

In our case we know the values for the Opposite and the Adjacent sides of the drift angle. This relationship:

$$\frac{\text{Opposite}}{\text{Adjacent}}$$

is known as the *Tangent* function ("Tan" for short). We can use the "Tan-1" button to find the angle. Using the calculator:

- 2/5 is 0.4
- Now press Tan-1

The calculator gives an answer of "21.8". For pilots 22° is close enough. Our error angle was 22°.

The last of the three trigonometry functions deals with the relationship:

$$\frac{\text{Opposite}}{\text{Hypotenuse}}$$

This is known as the *Sine* function ("Sin" for short).

One of these three functions, Cos, Tan or Sin should be enough for you to work out the angles or relationships of lengths for any triangle.

As with our earlier examples of equations, quite often there is no need to work out values at all. All we want to do is understand the relationships between things. You will see many examples during your ATPL course where relationships are described using one of these three functions.

Section 2

Equations of Trigonometry

2.2.1 Equations of Trigonometry

The only problem with the Sin, Cos and Tan functions is that you have to remember the relationships. No calculator will do this for you. Luckily there are two easy ways to remember them. The first is: "SOHCAHTOA" which phonetically sounds like "sow/ka/toe/ah".

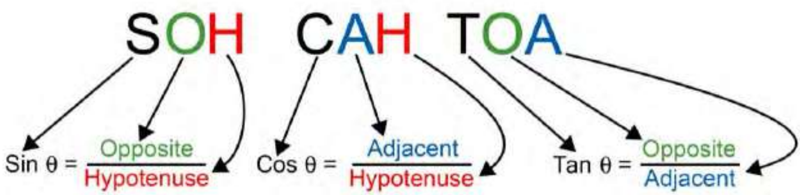


Figure 2.6 One method for remembering the three trigonometric functions

This word splits the functions into three parts as shown in Fig 2.6.
The alternative way of remembering the functions is the silly but memorable phrase "Tom's Old Aunt Sat On Her Coat And Hat".

$$\mathbf{T}\text{oms} = \frac{\mathbf{O}\text{ld}}{\mathbf{A}\text{unt}}$$

$$\mathbf{S}\text{at} = \frac{\mathbf{O}\text{n}}{\mathbf{H}\text{er}}$$

$$\mathbf{C}\text{oat} = \frac{\mathbf{A}\text{nd}}{\mathbf{H}\text{at}}$$

It doesn't matter which method you use to remember the relationships, but remember them you must!

Using a Calculator

2.3.1 Trigonometry and the Calculator

The trig functions are so important to understanding so many aspects of the course that it's worth recapping the calculations.

Finding an Angle When the Sides are Known

In the triangle in Fig 2.7, the Opposite side is 2 cm long and the Hypotenuse is 4 cm long. Find the angle theta.

Opposite over the *Hypotenuse* is governed by the Sine function.

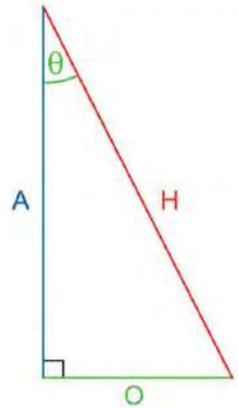


Figure 2.7 Using the Sine function

Therefore Sin θ =:

$$\frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{2}{4} = 0.5$$

On the calculator enter 0.5 then press Sin^{-1}. Answer: *30 degrees*.

Finding a Side Given one Side and one Angle

Let's look at a slightly more complicated case where we know the length of one side and we know the size of the angle. In this case we want to find out the length of the *Opposite side*.

Look at Figure 2.8. The angle is 20° and we have been told that the Hypotenuse is 5 cm long.

Firstly we need to chose the right trig function. The right one will be whichever formula mentions the value we already know and the side whose length we want to know. Only one of them does this, the Sine function:

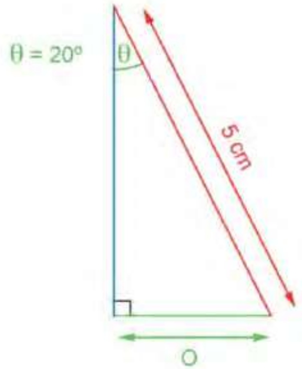


Figure 2.8

$$\text{Sine} = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

We know that our angle is 20° so we can substitute numbers for two of the three elements of this formula:

$$\text{Sine } 20^\circ = \frac{\text{Opposite}}{5}$$

We want to get the Opposite by itself by moving the "divide by 5" bit over to the other side of the equation. Move it across, remembering to change its sign from divide to multiply.

Now we have a formula which looks like this:

$5 \times \sin 20^\circ = \text{Opposite}$

Next we need to find the value for Sin 20°. Using your calculator enter 20 then Sin. The answer is 0.342.

Now our formula looks like this

$5 \times 0.342 = \text{Opposite}$

Answer: The length of the opposite side is *1.71 cm*.

Alternatively, if we had wanted to find the length of the Adjacent side we would have had to use the Cosine function because is the only one of the three which mentions the Adjacent and the Hypotenuse.

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\cos 20^\circ = \frac{\text{Adjacent}}{5}$$

$\cos 20^\circ \times 5 = \text{Adjacent}$

Cos 20° is 0.939

So $0.939 \times 5 = \text{Adjacent}$

The length of the adjacent side is 4.7 cm

2.3.2 Trigonometry in Aviation

If you read this chapter right through to the end then the chances are that you find it difficult to understand how, and why, maths is relevant to your pilot studies.

You may also be one of those people who need to understand the relevance of something before it becomes important enough to commit to memory!

We have already shown you how the Tan function can be useful in working out a drift angle. In the next chapter we start to use other trigonometry functions to resolve vector components - yet more vital basic knowledge for your ATPL studies.

3

Measuring Speed

Before we talk about the units, laws and formulae of physics we first need to understand one very vital point concerning frames of reference.

You can only measure the motion of an object relative to something else

Section 1

Frames of Reference

3.1.1 Frames of Reference

It's important to recognise that you can only measure the motion of an object *relative to something else*. Imagine you are doing a spacewalk, and the distance to your SpaceX Dragon capsule is increasing. Are you the one that's moving, or is it the shuttle? Relative to the Earth, you are both moving! There is no absolute frame of reference, only relative ones.This is both good and bad news.

The good news is that although you could understand a situation relative to any point, there is often one obvious frame of reference that makes a particular calculation or explanation easy. The bad news is that if you muddle up your frame of reference, you'll get confused.

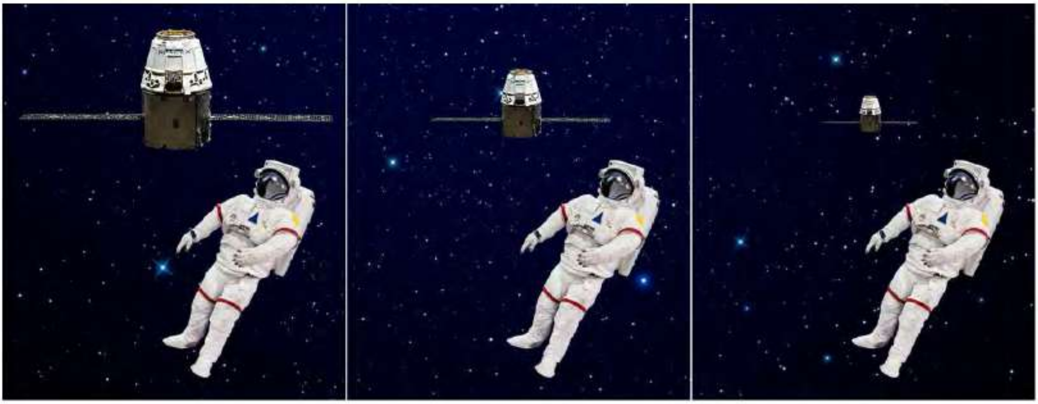


Figure 3.1 Who is moving - the astronaut or the SpaceX Dragon? Images courtesy of NASA and SpaceX

In *Principles of Flight* a number of different frames of reference can be used. Depending on the circumstances it may be convenient to reference the Earth, the air, the flight path, the aircraft or even the universe.

But whatever reference frame we choose, we need to define a point from which we can measure distances. We also need a set of axes to describe both the orientation and direction of travel.

Vector Quantities

3.2.1 Vectors and Scalars

As soon as we start considering an object in motion one thing we will probably want to know is which direction it is moving in. For example, even if we know where an object is, and the speed it's travelling at, we will not be able to predict its future position unless we know its *direction* of travel.

Vectors are used to describe both a *direction* and an *amount*. Quantities which have no specified direction are known as *scalars*. Vectors are frequently used in explanations of flight so it is essential to understand them.To illustrate the difference between vectors and scalars we'll use some examples of physical units, but we won't worry too much about their exact definitions until a little later.

When you push something, the direction it moves in depends on the direction in which you push. You can push it back to where it started by pushing in the opposite direction. Because we are describing direction, this implies that both the push and the resulting movement are vector quantities.

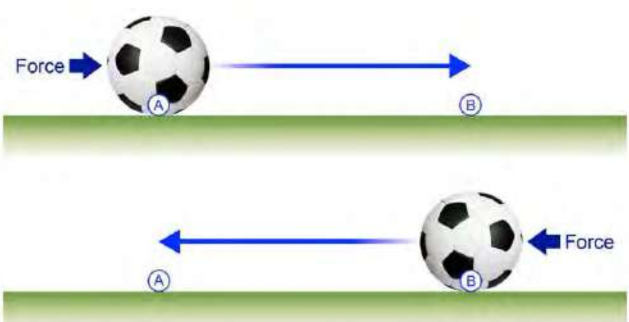


Figure 3.2 The push involves direction so it's a vector quantity

Now imagine you have a beaker of water and a source of heat. If you position the flame over the beaker, the water temperature will increase.

If you then apply the flame from the opposite side of the beaker the temperature will, of course keep increasing.

The heat we are adding, and the resulting temperature of the water, have no inherent direction. If they did, then we ought to be able to cool the water down by applying the flame from the opposite side! Clearly this won't happen, so the heat and the resulting temperature are both scalar quantities.

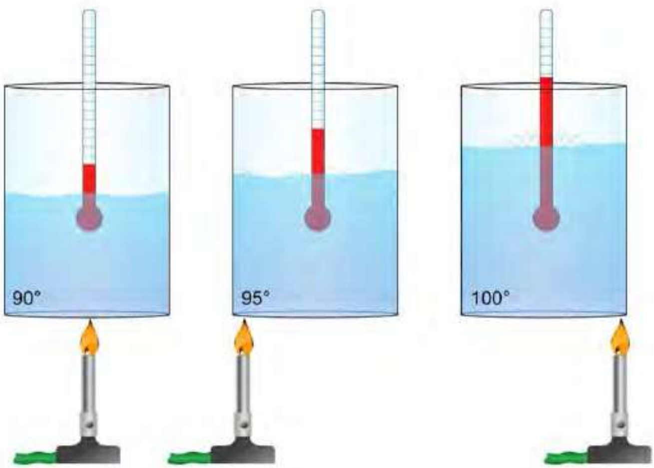


Figure 3.3 Temperature has no direction so its a scalar quantity

3.2.2 Adding Vectors

Vector quantities cannot be added together using simple arithmetic. For example, if you start from Point A and walk 10 metres East (090°) to Point B, followed by 20 metres South West (225°) to Point C you will not be able to work out where you are just by adding 10 plus 20. To solve the problem you have to use geometry - and more specifically, vectors (Figure 3.4).

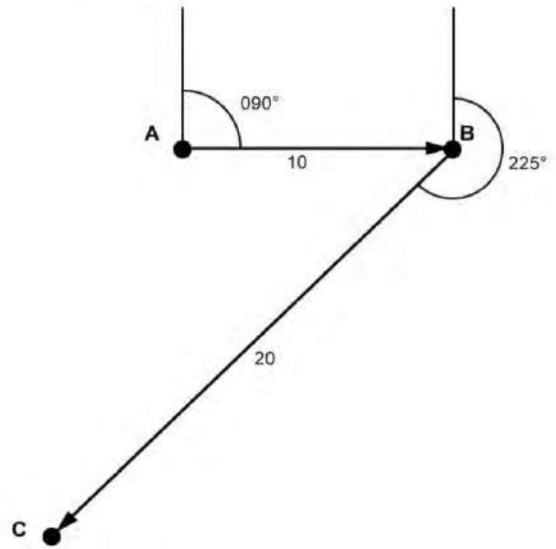


Figure 3.4 Vectors show quantity and orientation

The vector is depicted as an arrow; the length of the arrow represents the amount of the quantity. The orientation of the arrow represents the direction of the effect.

The combined effect of two vectors can then be seen by adding them together 'tip to tail'. You then complete a triangle by placing a third vector from point A direct to point C. The length of this vector tells us how far the end point is from the start point; and of course the orientation tells us the overall direction travelled.

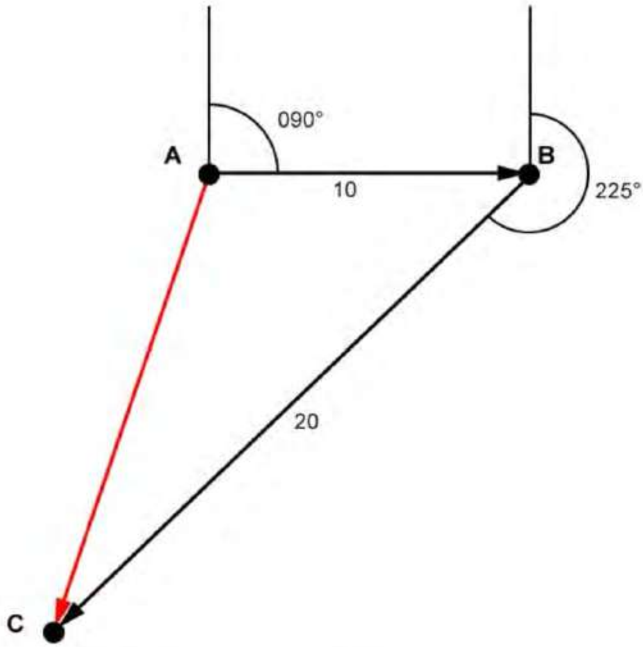


Figure 3.5 Two or more vectors can be added together

You can do this either by scale drawing, or by using appropriate mathematics.

There is a very common aviation application for this; adding a wind vector (wind direction and speed) to an air vector (heading and true air speed) to calculate a ground vector (track and ground speed). When you use the CRP-5 navigation computer you are solving this calculation by scale drawing.

The CRP-5 is essentially a reusable piece of graph-paper with a protractor and ruler built in.

We can use the principle of vector addition to see the combined effects of several influences acting on an object.

3.2.3 Resolving Vectors

If you can add two vectors together to create a single vector showing the combined effect, then you can also do the same thing in reverse. In other words you can split a single vector into its two component vectors.

For example in this diagram we can see that the original vector could be replaced by all sorts of combinations of other vectors. Most commonly vectors are broken into two components at right angles to each other and aligned with the system of axes (using whatever frame of reference we have chosen).

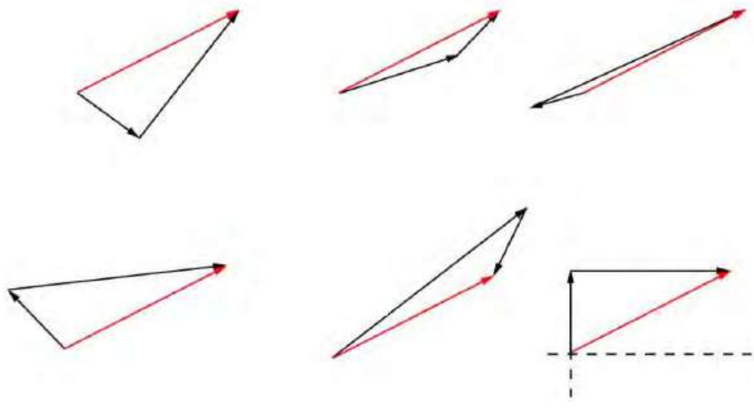


Figure 3.6 Resolving one vector into its two component vectors

This can be done by scale drawing. Or, where our axes are at right angles to one another, by calculation. This is an important point: for all triangles which contain a right angle we can easily calculate the lengths of the sides (vectors) using trigonometry. If you understand it then the next bit will be easy. If not, then read the Trigonometry chapter again or learn the next bit off by heart.

Imagine we have two axes, X and Y as shown below, and a vector V that we wish to resolve into two components (horizontal and vertical).

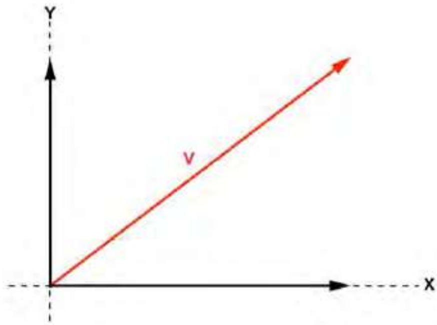


Figure 3.7 Finding the value of Vector "V" by calculation

To do this, we have to know the orientation of the vector. Let say that we know the direction it is pointing, as measured from the Y axis, and we will call this angle θ (the Greek letter theta). If we slide the X component upwards, we can create a right angled triangle which splits vector V into its X component and Y component.

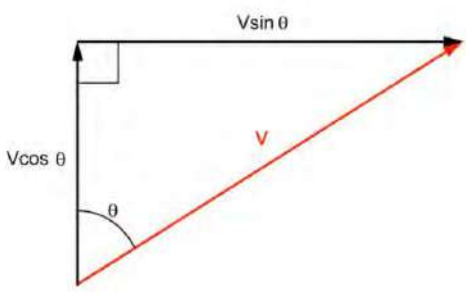


Figure 3.8 Creating a right angled triangle

We can now use trigonometry to calculate the following:

The X component = V x Sin θ

The Y component = V x Cos θ

You must use Cosine for the vector component which acts along whichever axis you have used to define the orientation of the vector.

"It's COS be*cause* it's next to the angle."

Whenever you see a vector, you can drop a pair of right angle axes onto the vector and resolve the components.

Which Quantities are Vectors?

It is usually fairly easy to see if a quantity is a vector or a scalar but there are some quantities where the distinction is quite tricky. So, if it's easy, we'll tell you and explain it. If it's difficult but helpful, we'll just tell you but if it's difficult and not relevant we won't even mention it.

Units of Measurement

3.3.1 Time

The SI unit for time is the second. In time calculations be careful to convert everything to the same unit.

For example 1 minute 15 seconds should be converted to 75 seconds. (60 seconds for the minute plus the additional 15 seconds).

3.3.2 Describing Sizes

Length (L)

We can measure something in one dimension, and this is its Length in metres. Distance is the same concept – the length between two points.

We can use shorthand methods for all of these terms; e.g. Length is often abbreviated to L and Distance to D. Likewise, the SI unit has a shorthand symbol, in this case m for metres.

Area (A)

Area is the two dimensional size of a defined surface, measured in square metres (m^2).

Volume (V)

Volume is the three dimensional size of an object, measured in cubic metres (m^3).

3.3.3 Interim Summary - Units of Measurement

CONCEPTS		UNITS	
NAME	SYMBOL	NAME	SYMBOL
Time	T	seconds	s
Length, or distance between two points	L	metres	m
Area	A	squared metres	m^2
Volume	V	cubic metres	m^3

One-Dimensional Motion

3.4.1 Speed

We can describe how rapidly something is moving with the everyday term speed, measured in metres per second (m/s). You can think of speed as the *rate at which distance is changing*.

Recall the formula for speed:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

There is a handy clue there, if you rewrite the formula in units you get:

$$\text{Metres per second} = \frac{\text{Metres}}{\text{Seconds}}$$

Divided by is the same as saying '*per*'. So, as long as you can remember any speed unit (for example *miles per hour*) then it is easy to see that a speed is calculated by taking a distance unit and dividing it by a time unit.

3.4.2 Acceleration

We also need to be able to describe changes in the way an object is moving. We use the term acceleration to describe this. It's possible to get a bit confused about this, so we will start by looking at a one-dimensional situation; motion along a line.

In this context we use the term acceleration to describe how rapidly a speed change occurs. The units are metres per second per second. This is sometimes known as metres per second squared or just m/s^2.

In the same way that speed is the rate at which distance changes, acceleration is the rate at which speed changes.

You can calculate the acceleration of an object by looking at the change in speed that occurs over a known period of time, and then dividing it by the time that has passed.

Example: a man is running at 5 metres per second. The man accelerates and 5 seconds later he is is travelling at 15 metres per second. What is the acceleration?

Acceleration = Change in speed / time

Acceleration = 10/5 = 2m/s^2

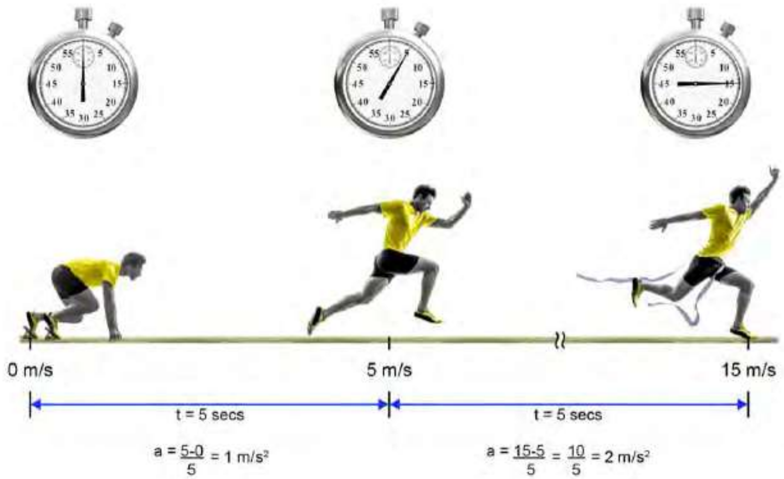


Figure 3.9 Measuring acceleration

Obviously, this figure gives an average acceleration. In reality, the car's acceleration would decrease as it increases speed and would occasional dip as the driver changed gear.

It is easy to get confused between change in speed and acceleration. In everyday speech our runner might be described as having accelerated by 10 metres per second.

More correctly, it has *changed speed* by +10 m/s, and has accelerated at + 2 m/s^2.

Similarly if you know an object's acceleration and starting speed you can calculate its speed in the future.

Imagine a Dragon capsule is motionless having just undocked from the international space station. It burns its engines and accelerates at 5 m/s^2. After one second it will be moving at 5 m/s. Every second that passes will see it moving another 5 m/s faster; after 10 seconds it will be moving at 50 m/s; all relative to the space station of course.

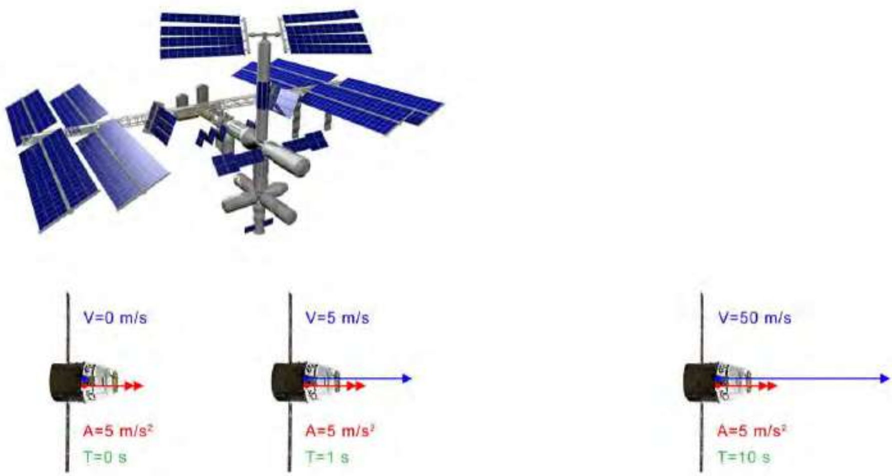


Figure 3.10 Relative acceleration

3.4.3 Graphs of Motion

If you have a graph showing an object's speed versus time, then the acceleration at any particular time is shown by the steepness of the slope of the graph at that point.

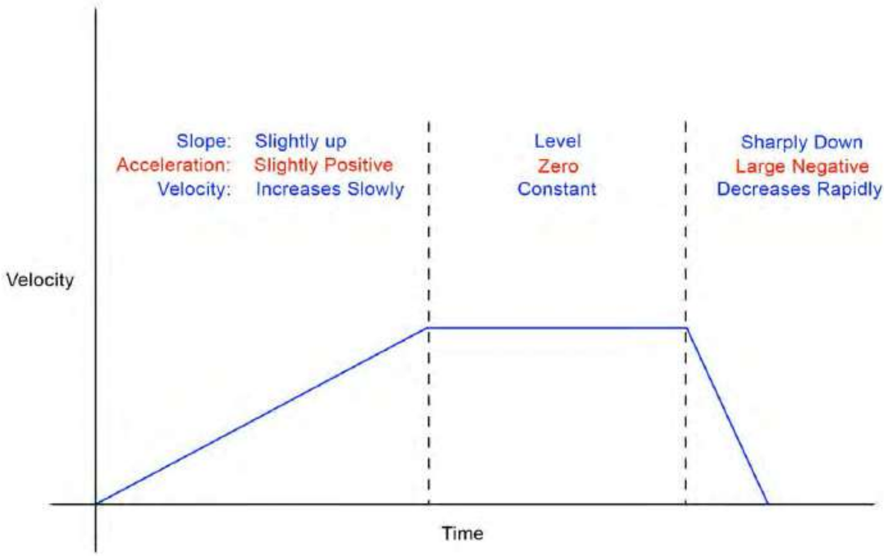


Figure 3.11 Acceleration is shown by the steepness of the slope

2 or 3 Dimensional Motion

More complex situations involve two or more dimensions. We've not yet considered whether or not speed is a vector. In fact speed is a scalar. There is a vector equivalent however, and it is known as *velocity*.

3.5.1 Velocity

An object's velocity vector has a magnitude (length) equal to its speed, but it also describes the direction in which the object is moving.

Look at the following diagram, which shows the flight path of an aircraft flying a turn. At each point in the turn the arrows represent the velocity vector of the aircraft. Notice that the arrows are all the same length, so the aircraft is flying a constant speed. However, the direction of motion and therefore the velocity is continually changing.

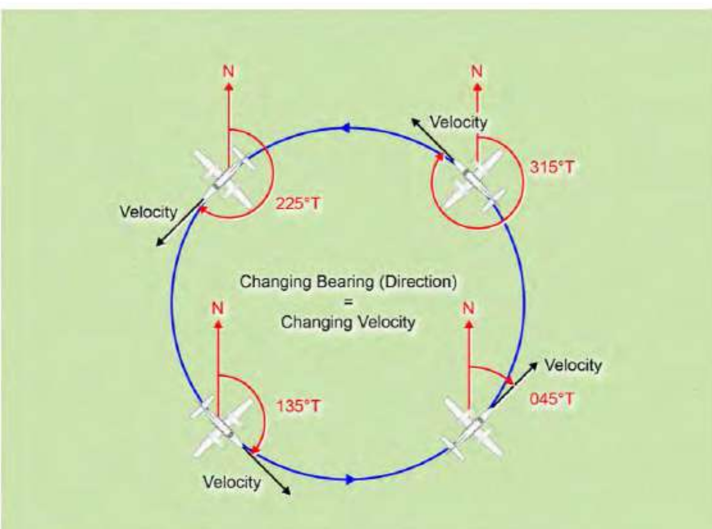


Figure 3.12 Constant speed but changing velocity

3.5.2 Acceleration

When we first looked at acceleration we considered a 1-dimensional example. But now we need to be more precise and state that acceleration relates to changes in velocity, not just changes in speed.

$$\text{Acceleration} = \frac{\text{Change of Velocity}}{\text{Time}}$$

Whenever you see a change in direction there must be at least some amount of acceleration at right angles to the direction of travel. In the case of our aircraft, the only thing that's changing is the direction, so the acceleration is all at right angles.

In the case of circular motion, as in our turning example, the object is always accelerating at right angles to the direction of travel, inwards towards the centre of the circle.

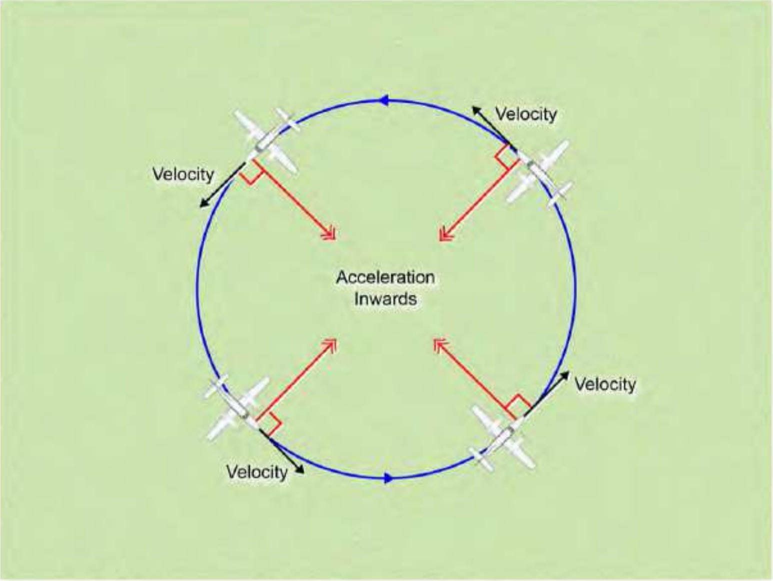


Figure 3.13 Change in velocity means the object is accelerating

Because acceleration depends on the direction of the velocity vector, it should come as no surprise that acceleration is also a vector quantity.

It is possible to calculate the size of acceleration involved. It is related to the speed that the object is moving at, and the radius of the circular path being followed. The maths behind this is relatively complex and beyond the scope of the course, however the end result is worth knowing:

$$\text{Acceleration} = \frac{\text{Velocity squared}}{\text{Radius}}$$

$A = V^2/R$

It's worth recognising the velocity term is squared. So, if you double the velocity you need 4 times the acceleration to follow the same path. Or, if you maintain the same acceleration, the radius of the turn will increase by a factor of 4.

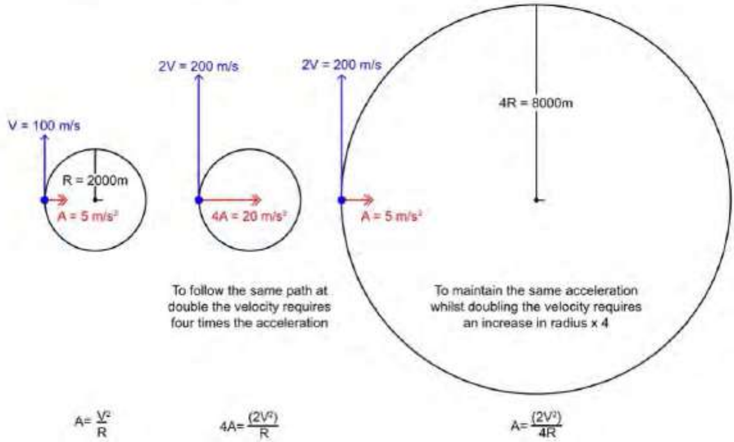


Figure 3.14 Acceleration varies with the square of the velocity

From a practical point of view the implications are clear. If you are trying to follow a particular flight path, for example a published approach procedure, it is critical that any published speed limit is followed. If you exceed the design speed for the procedure, your increased radius of turn may take you outside the protected airspace.

3.5.3 Summary

CONCEPTS		UNITS	
NAME	SYMBOL	NAME	SYMBOL
Velocity	V	metres per second	m/s
Rate of change of distance, a vector.			
Speed	S	metres per second	m/s
The scalar equivalent of velocity but with no associated direction.			
Acceleration	A	metres per squared second	m/s^2
Rate of change of Velocity, a vector.			

4

The Laws of Motion

So far we have looked at terminology to describe how things are moving, and how that movement might change. The next area to study is why things move.

Sir Isaac Newton established a set of principles to explain this area of physics; these are known as Newton's laws of motion.

Section 1

Newton's Laws

Sir Isaac Newton established a set of principles known as "*Newton's laws of motion*".

Although the physics behind them is actually fairly easy confusions do arise. The basic problem is that in everyday life we are used to the idea that you have to keep pushing something to keep it moving. This makes people think that any moving object has some sort of inherent 'push' acting in the direction it's moving. In fact this is not the case. The first step in seeing what's actually happening is to introduce a new quantity: *force*.

4.1.1 Force

A force is anything that causes a 'push' or a 'pull' effect. The unit of force is the *Newton*. As we saw earlier, the direction that you push or pull something is important, so force is a vector quantity. We use the letter F as an abbreviation for force, and N for Newtons.

The combined effect of several forces acting on a single object can be figured out by adding the forces together as vectors.

For example, consider a tug of war.

Team A is pulling to the left with a force of 10 000 Newtons, whilst Team B pulls to the right with a force of 10 200 Newtons, the combined effect is a pull to the right of 200 Newtons. Team B will win.

The combined effect of several forces acting together is known as the *resultant force*. If, after adding all the forces together as vectors, there is an overall push in one direction, this resultant is known as an *unbalanced force*.

In our example the unbalanced force is 200 Newtons.

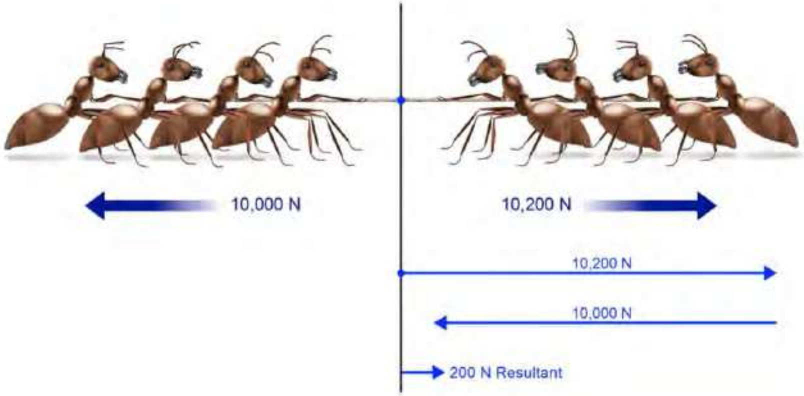


Figure 4.1 An unbalanced force (Courtesy Shutterstock)

It's the size of the unbalanced force that determines what happens to the movement of an object. If both our teams became tired and reduced their pulls by 1 000 Newtons each, the unbalanced force would still be 200 Newtons. Team B will still win.

Now that we've established what a force is, and how several forces can be added up, let's consider how they relate to movement.

We are used to the idea that a stationary object will remain stationary until we push or pull it, i.e. until we apply an *unbalanced* force.

However, Newton also established the idea that moving objects will continue to move with the same velocity unless they are subject to an unbalanced force.

At first sight this conflicts with our everyday experience. When you push something, like a car, you know you have to keep pushing to keep it moving.

Figure 4.2 A continuous push is required to balance the frictional forces trying to slow the car

But this is not the complete picture. When you cease to push, the car slows down and stops not because it has an inherent tendency to be motionless, but because there are frictional forces at work slowing it down. At walking speeds the opposing force is provided mostly by friction in the wheel bearings and the contact between the tyres and the road.

When you push the car at a constant speed, your pushing force balances the frictional forces. The resultant force is zero so the car continues at the same velocity.

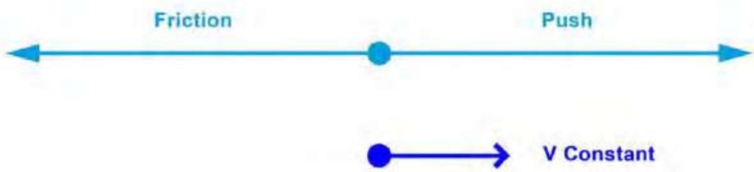


Figure 4.3 The car example shown as force vectors

If you stop pushing, the car has an unbalanced frictional force that slows it down. In other words, objects slow down because of the presence, not absence, of a force.

Let's simplify things, and find a place where we don't have to worry about friction. Imagine that we go deep into space, far away from any stars or planets so that the effect of gravity can be ignored. We are in a vacuum so there will be no air resistance, in other words we have removed friction.

Now imagine that we have an astronaut, doing a spacewalk. She throws a spanner away from her, giving it a velocity of 1 metre per second. Now we will see the true nature of motion, we'll take the astronaut as the point of reference.

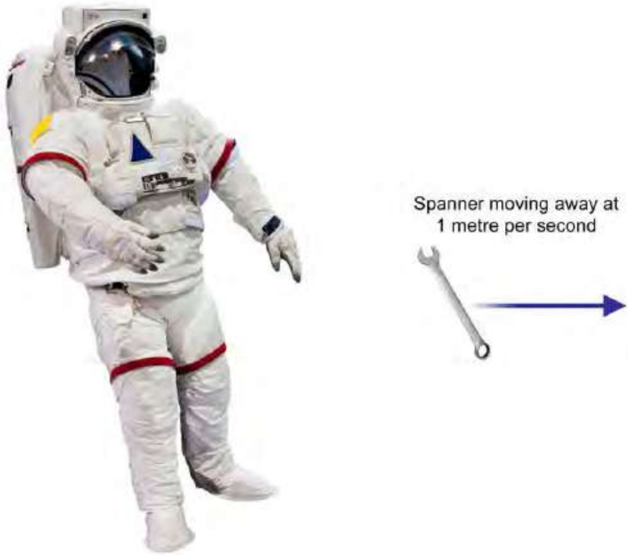


Figure 4.4 Giving the spanner an initial velocity......

One minute later the spanner will be 60 metres away, still moving at 1 metre per second. One hour later it will be 3 600 metres away, and it will still be moving at 1 metre per second. One year later the spanner will still be moving away at 1 metre per second.

There are no unbalanced forces, in fact no forces at all in this example, so the spanner stays at the same velocity.

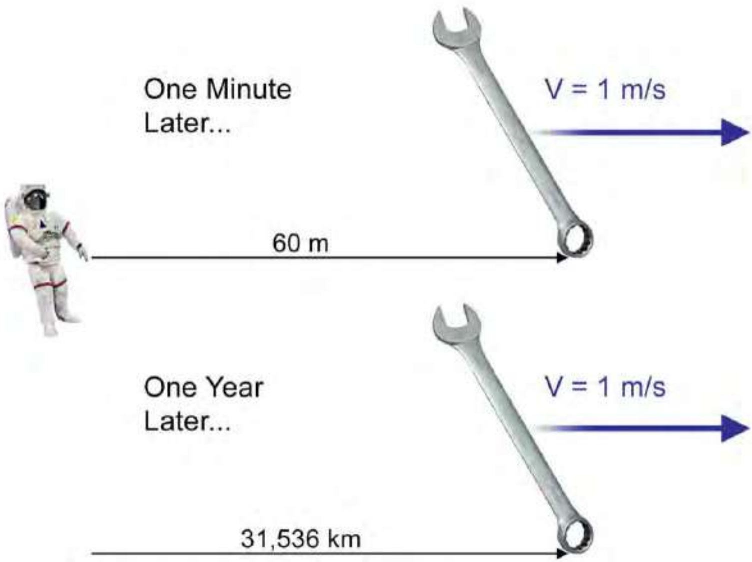


Figure 4.5 ....which remains unchanged for eternity

Section 2

Newton's 1st Law

4.2.1 Newton's 1st Law

An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force

This is the formal statement of the law, but we can simplify it. The phrase "motion with the same speed and in the same direction" simply means constant velocity. Additionally, being 'at rest' means velocity is zero and remaining zero. It's just another example of constant velocity. So Newton's first law boils down to this:

The velocity of an object will be constant unless it is acted upon by an unbalanced force

So, if an object has constant velocity, and hence an acceleration of zero, we can say with certainty that one of two conditions exists. Either:

- The object has *no* forces acting on it at all; or
- The object has *no unbalanced forces*, i.e. the vector addition of all forces is zero.

It's implicit in Newton's first law that if there is an unbalanced force acting on an object, then the velocity will change; in other words there will be an acceleration.

Our next step is to study how *much* acceleration will be caused by an unbalanced force. Newton considered this in the second of his laws.

Section 3

Newton's 2nd Law

4.3.1 Newton's 2nd Law

We can't begin this section with a formal definition of Newton's second law, because we haven't yet covered all the necessary terminology. We must build towards it and introduce the terminology as we go.

We've seen that an unbalanced force will cause an acceleration. The question is, how much? It should be reasonably obvious that a small unbalanced force will have a small effect, and that a larger force will have a greater effect.

In mathematical terms the acceleration is directly proportional to the unbalanced force. So, if everything else is fixed:

Acceleration $\propto$ Force

Recall that proportional means:

- Double the force - double the acceleration
- Triple the force - triple the acceleration
- Half the force - half the acceleration

Yet different objects subjected to the same sized force will not necessarily experience the same acceleration. Some objects seem more reluctant to be accelerated than others.

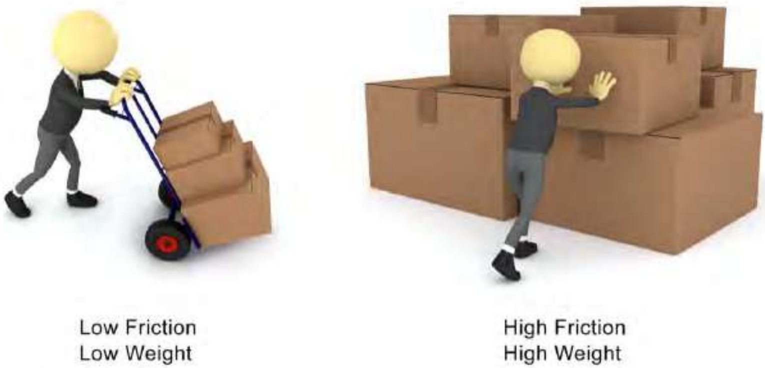


Figure 4.6 Some objects are more reluctant to be accelerated than others (image portions Courtesy Shutterstock)

4.3.2 Mass

Mass is the name we commonly use to describe the amount of matter contained in an object. Most commonly we measure mass by the weight force it produces in Earth's gravity.

But in the context of Newton's laws of motion it is more useful to define mass as the reluctance of an object to be accelerated (this is the definition of *inertial mass*).

Mass (M) is measured in kilograms (kg). The more mass an object has, the less it will accelerate in response to a given force. This relationship is inversely proportional. Assuming a fixed force:

$$\text{Acceleration} \propto \frac{1}{\text{Mass}}$$

Inversely proportional means:

- Double the mass - half the acceleration.
- Triple the mass - one third the acceleration.
- Half the mass - double the acceleration.

We can merge these two relationships together as follows:

$$\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}$$

This is one mathematical way of expressing Newton's second law:

$$\text{Acceleration}\,(\text{m/s}^2) = \frac{\text{Force(N)}}{\text{Mass(kg)}}$$

This formula shows us that the acceleration caused by a force:

- Increases when the force get bigger (the F is on the top of the equation)
- Increases when the mass gets smaller (the M is on the bottom of the equation).

If we do the maths we can see that an acceleration of 1 metre per second squared is caused when an unbalanced force of 1 Newton is applied to a mass of 1 kilogram.

We can also rearrange the formula like so:

$F = MA$

Force = Mass x Acceleration

This is another common way of expressing Newton's Second Law. It's the same relationship, just displayed differently. If we know the acceleration an object is experiencing, and we know the mass of the object, then the unbalanced force that caused it can be calculated.

The formula can also be expressed in terms of Mass:

$$\text{Mass(M)} = \frac{\text{Force(F)}}{\text{Acceleration(A)}}$$

So, if you know the acceleration that an object achieved, and you know the force that was applied, you can calculate the mass of the object.

So, to summarise, we now have a basic three-term formula relating Force, Mass and Acceleration. We've seen that this relationship can be expressed in a number of ways but we still haven't yet reached a formal statement of Newton's second law.

4.3.3 Mass or Weight?

Before we go further we need to dispel one common confusion: that *mass* is the same as *weight*.

In everyday language, we use the word "weight" imprecisely. In physics weight refers to the pull on an object due to gravity. We've already introduced a term for 'push' or 'pull' effects; *force*. So an object's weight is the *gravitational force* acting upon it, and should correctly be expressed in terms of Newtons rather than kilograms.

Gravity is such a weak force we don't feel ourselves pulled towards other objects. The only object big enough for us to sense the pull of gravity is the Earth.

The gravitational effect is proportional to the mass of our planet (which is effectively constant) and the mass of the object concerned.

In other words each planet or moon has its own gravitational field strength. An object has a weight (in Newtons) proportional to its mass in kilograms. For the Earth, the field is approximately 9.82 Newtons per kilogram. Hence:

Weight (N) = Field Strength (N/kg) x mass (kg)

For the Earth:

Weight (N) = 9.82 (N/kg) x Mass (kg)

This is often approximated to 10 Newtons per kilogram.

Exam Questions

When answering an exam question always check carefully to see if it tells you what value to use; if in doubt use 9.82. Similarly don't confuse yourself by answering with the wrong units. For example:

The weight of a 70 kg box of cargo is 700 Newtons.

The mass of the same box is 70 kg.

4.3.4 Acceleration Due to Gravity

If we apply Newton's second law to a falling object, then we get an interesting result. If an object has more mass, then it should be more reluctant to accelerate. So, at first glance massive objects should fall more slowly than less massive ones!

But more mass means more weight so these two considerations cancel one another out. On Earth every kilogram of mass has 9.82 Newtons of weight to accelerate it, and so every object accelerates at 9.82 m/s^2. Put simply, all objects accelerate at the same rate.

This appears to fly in the face of our everyday experience because we know, for example, that a hammer falls faster than a feather. Yet again it is frictional forces which confuse the issue. Objects falling through air experience air resistance which opposes their acceleration towards the Earth.

The resultant force on the object is the vector addition of the weight and the air resistance. As the object falls faster, air resistance increases and the resultant force on the object gets smaller. Consequently the acceleration decreases.

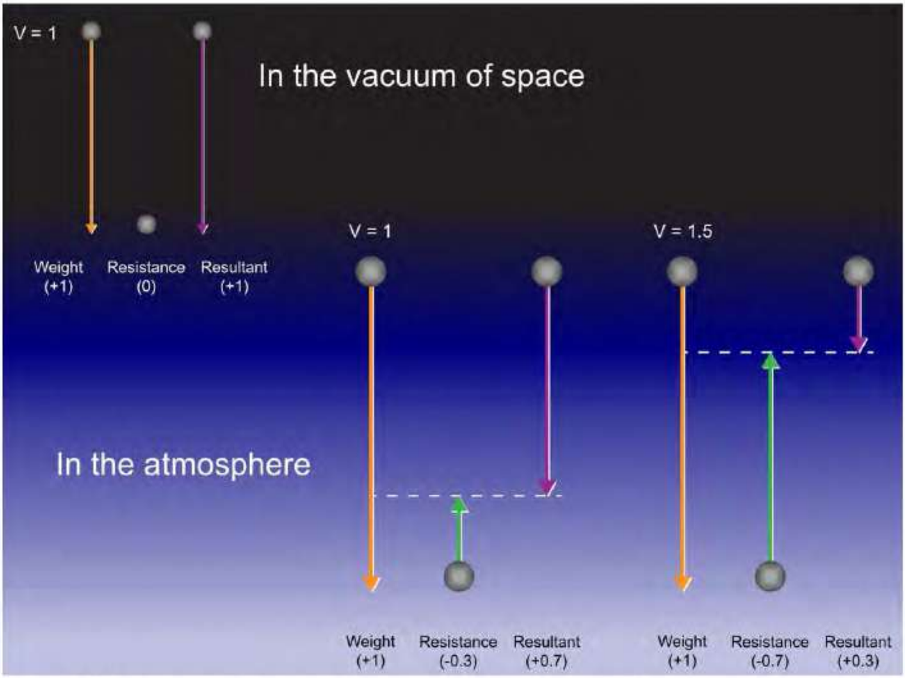


Figure 4.7 In the Earth's atmosphere movement is opposed by air resistance

Terminal Velocity

Eventually an equilibrium is reached when the air resistance becomes exactly equal and opposite to the weight. At this point acceleration is zero and the object has reached its *terminal velocity*.

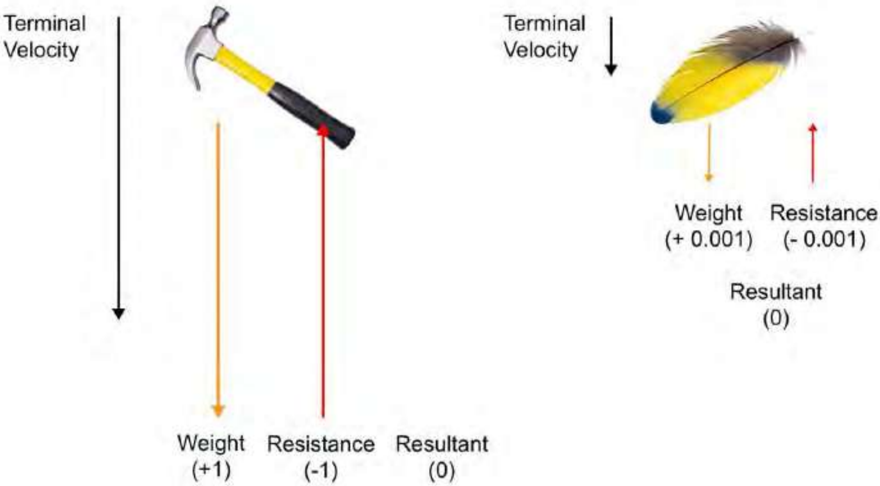


Figure 4.8 Terminal velocity (images courtesy Shutterstock)

Terminal velocity differs for different objects. If we drop a heavy object, for example a hammer, alongside a lightweight object like a feather, then air resistance fundamentally affects the terminal velocity.

For the hammer, the air resistance is quite small relative to the weight force, so the hammer has to fall very fast before it reaches equilibrium. On the other hand a feather has very little weight force driving it downwards so air resistance easily opposes it and terminal velocity is reached almost immediately.

In an atmosphere, heavier objects tend to have higher terminal velocities, even though all objects have the same initial acceleration.

The most convincing demonstration of this phenomenon was given by astronaut Dave Scott at Hadley Rille on the moon during the Apollo 15 mission.

When Scott simultaneously dropped a hammer and a feather both objects hit the ground at the same time, clearly demonstrating that the acceleration due to gravity does not depend on the mass of an object.

Movie 4.1 Commander Dave Scott demonstrates that a hammer and feather accelerate towards the surface of the moon at the same rate (courtesy NASA)

4.3.5 Weight and Acceleration

Because Newton's second law is often quoted, and remembered, as F = ma, it is possible to think that the formula implies some sort of cause and effect but it doesn't. Newton's second law simply observes that force and mass times acceleration are mathematically equivalent. If you know the mass and the acceleration you can calculate the force which produced the acceleration or, if you know the force, you can calculate the acceleration that will be given to a mass.

Hopefully you now understand the difference between weight and mass. As a final example consider a 1 kg dumbbell. It has a *mass* of 1 kg wherever it is. But its *weight* will depend on *where* it is.

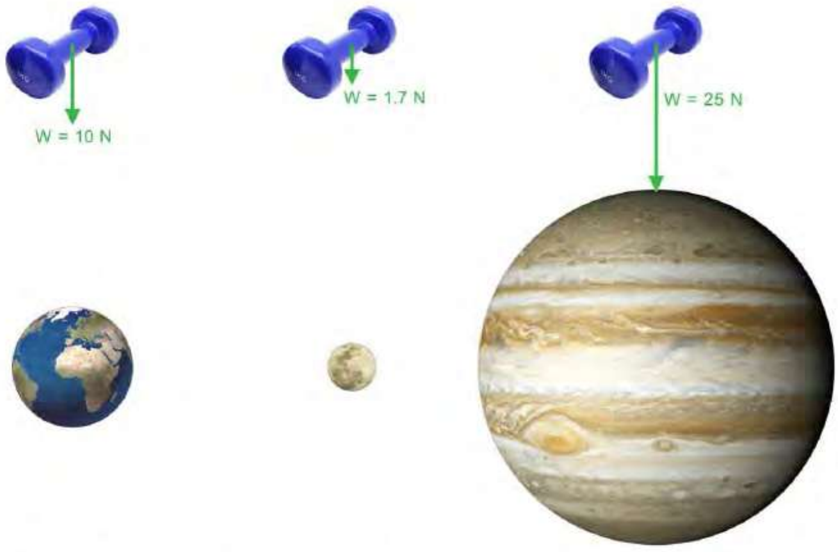


Figure 4.9 Weight depends on the gravitational field

Because weight is a force, it is a vector. Mass is a directionless scalar.

4.3.6 Momentum

It's often very useful to keep track of the value of an object's mass multiplied by its velocity. In fact it's so useful, scientists have given the resulting quantity its own name: *momentum*.

Momentum = Mass x Velocity

Momentum has the symbol p, although we don't often need it. The SI units are expressed as kilogram metres per second (i.e. a mass unit multiplied by a velocity unit). Once again, direction is important, so momentum is a vector quantity.

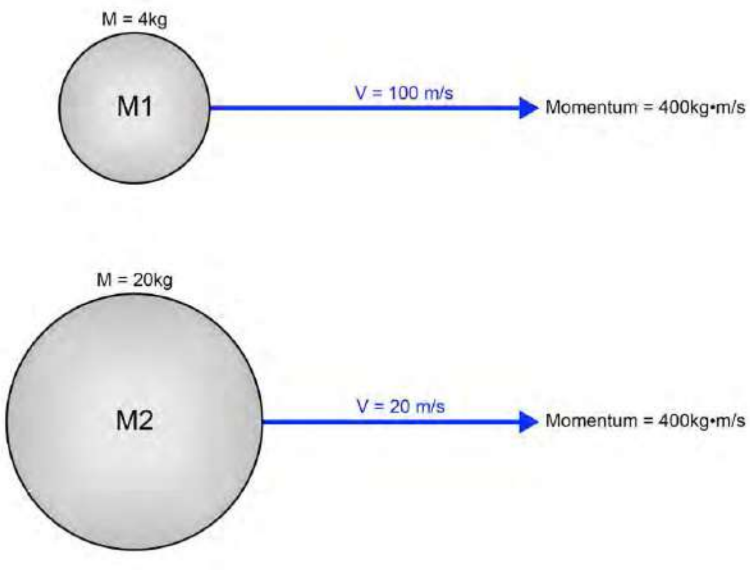


Figure 4.10 Momentum is the product of mass and velocity

Note that we can have a particular amount of momentum from a small mass moving fast, or a large mass moving more slowly.

Momentum is a useful quantity because it accounts for the effects of forces in a system over a period of time. We can use maths demonstrate this. If it looks too daunting skip the next page.
Momentum = Mass x Velocity

Divide both sides by Time:

$$\frac{\text{Momentum}}{\text{Time}} = \frac{\text{Mass x Velocity}}{\text{Time}}$$

Rearrange slightly, for our convenience:

$$\frac{\text{Momentum}}{\text{Time}} = \text{Mass x } \frac{\text{Velocity}}{\text{Time}}$$

For an object with fixed mass, any change in momentum must be due to a change in velocity.

$$\frac{\text{Change in Momentum}}{\text{Time}} = \frac{\text{Mass x Change in Velocity}}{\text{Time}}$$

Substitute acceleration for change in velocity/time:

$$\frac{\text{Change in Momentum}}{\text{Time}} = \text{Mass x Acceleration}$$

Substitute force for Mass x Acceleration:

$$\frac{\text{Change in Momentum}}{\text{Time}} = \text{Force}$$

Change in momentum divided by time is the same as saying the 'rate of change of momentum'. So:

Rate of change of momentum = Force

So, a certain size force will cause a certain rate of change of an object's momentum. If an object has a lot of mass then a certain amount of force will only change the velocity of the object slowly. This is simply another way of stating the underlying concept of Newton's second law.

4.3.7 Newton's 2nd Law Defined

We have finally arrived at the formal definition usually quoted for this law:

The rate of change of momentum of an object is equal to the applied force.

It's worth remembering this statement as well as the more useful equation forms of Newton's second law that we saw earlier.

Newton's second law has an important implication. Because force is equal to the *rate* of change of momentum, if we need to cause a particular change in momentum we must apply a force for a certain amount of time.

For example, imagine we have a 10 kg mass moving at 5 metres per second. That's a momentum of 50 kg m/s. If we want to bring that object to rest, and we know the size of the force, then we can calculate how long this will take.

Important: momentum based calculations allow you to find out how much time it takes for a force to cause a particular velocity change on an object

4.3.8 Summary

CONCEPTS		UNITS	
NAME	SYMBOL	NAME	SYMBOL
Mass	M	kilograms	kg
A measure of how difficult it is for a particular force to accelerate an object. A scalar.			
Weight	W	Newtons	N
A force due to gravitational attraction. 9.82 Newtons per kilogram (on the Earth). A vector.			
Momentum	p	kilogram metres per second	kg.m/s
p = mv, A vector, always conserved			

Figure 4.11 Summary

Newton's 3rd Law

4.4.1 Newton's Third Law

Newton's third law is a much easier concept to understand, so we can go straight to it.

Every action has an equal and opposite reaction.

Newton's third law underpins almost every aspect of how we interact with the world around us. From a principles of flight perspective it underpins our understanding of lift and thrust. The words 'action' and 'reaction' can also be read as 'force'. The law tells us that every force has an equal size force acting in the opposite direction.

Remember, a force is a 'push' or a 'pull'.

Newton's third law tells us that if an object is being pulled (or pushed) in one direction, something else, somewhere in the universe, is being pulled (or pushed) in the opposite direction. This always applies; but sometimes you have to look quite carefully to spot the other object.

One simple example is the recoil from firing a gun. If the shell is being forced in one direction then the gun must be forced in the opposite direction.

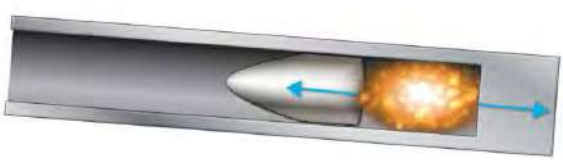


Figure 4.12 Equal and opposite forces are applied to both gun and bullet

Common Misconception

As we saw earlier, if an object has constant velocity this means the resultant force acting upon it is zero. In many simple situations this means that there are two forces acting, and they are equal and opposite.

For example, imagine a 1 kg house brick placed stationary on the ground. We know the brick has weight, of approximately 10 Newtons due to gravity. We also know that the brick isn't going anywhere; it has a constant velocity (zero), and is therefore not accelerating. Therefore it must have a resultant force of zero, so there must be at least one more force present. In this case it is an upwards force from the ground, and it must also be 10 Newtons.

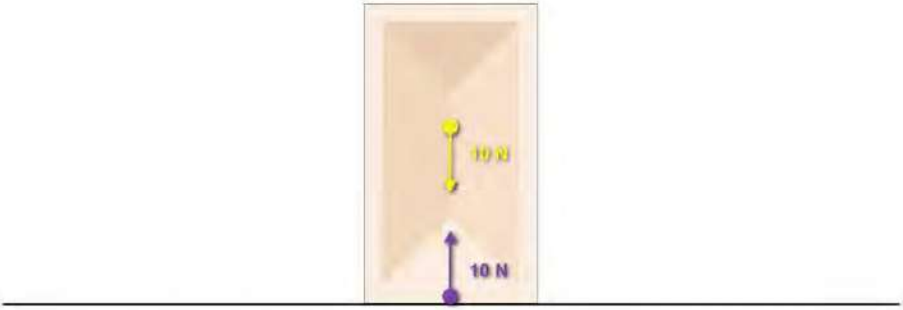


Figure 4.13 The equal and opposite forces acting on a brick

If you find the upwards force difficult to visualise, imagine if you were holding the brick on your palm, at arm's length. You would definitely feel the effort of holding the brick up.

So, we have two "equal and opposite" forces. Since the phrase "equal and opposite" also exists in Newton's third law, people often get the two concepts muddled up. The two forces we are looking at are not the equal action and opposite reaction. They just happen to be equal and opposite. This is in fact a case of Newton's first law where the resultant of two forces equals zero.

In Third law terms, the equal and opposite reaction to the weight of the brick is the fact that Earth also experiences a gravitational attraction to the brick. Of course, since the Earth has so much mass, it is the brick that will do the moving if it is dropped from a height.

The equal and opposite force to the reaction from the ground acting on the brick is that the brick also pushes down on the Earth. We have equal force pairs due to Newton's 3rd, and coincidentally the two pairs also happen to be equal to one another due to Newton's 2nd."

Note that there are no preconditions stated for Newton's third law. It doesn't say "every action has an equal and opposite reaction as long as the object is not accelerating". It just says "every action has an equal and opposite reaction", full stop, always.

4.4.2 Newton's 3rd Law and the Nature of Momentum

We need to understand the 3rd law relates to the concept of momentum. We'll start with two facts that we have already established:

- Objects with resultant forces that are not zero will undergo a rate of change of momentum equal to the force.
- Every force has an equal and opposite force.

When these two things are considered together, we can see that if one object undergoes a particular change in momentum, then there must be, somewhere else in the universe, something else that has an equal and opposite change of momentum. In other words, the total momentum involved in the system has not changed.

- Momentum is conservative. In every interaction between objects, momentum is conserved.

Remember, momentum is a vector quantity, so one object gains momentum in one direction, and another object has gained an equal and opposite amount of momentum, then the overall momentum change is zero. In Physics, when we have any quantity that exhibits this sort of behaviour, we refer to it as being conservative.

- In every interaction involving changes of momentum the total momentum remains unchanged.

5

Work and Energy

There are a complex system of interactions between objects. Whenever something changes it is because another object has done work on it.

Section 1

Energy

5.1.1 Introduction

When we look at the physical world around us we see a complex system of interactions between objects. Whenever something changes it is because another object has done some work on it. For example, if you press the accelerator in your car the engine does work on the body of the car to increase its velocity.

5.1.2 Energy

The ability of a system to do work is known as Energy (E). Energy cannot be created or destroyed, only changed from one form to another. In other words Energy is a conservative quantity, just like momentum.

Generally, whenever you see something changing in the world around you, energy is being changed from one form to another. Some part of the system is doing work on some other part of the system.

The unit for energy is the Joule (J), and it is a scalar quantity.

There are several different forms of Energy which we will introduce throughout the rest of this chapter. Those which are especially important to our studies will get a detailed treatment, others will be mentioned briefly.

Section 2

Kinetic Energy

5.2.1 Kinetic Energy

The energy of a moving object is known as its kinetic energy. If you want to make an object move, you have to do some work on it to speed it up. If you want to get the object to slow down, you need a way of getting it to do work on something else, for example by using a brake.

The kinetic energy (K.E.) of a moving object can be calculated by using the following formula:

$K.E = 1/2\ MV^2$

Where:

M = the mass of the object

V = the velocity of the object

It's worth noting that the velocity term is squared. This means that doubling the velocity requires four times the energy; even in a frictionless situation.

Section 3

Heat and Temperature

5.3.1 Heat and Temperature

Heat is a variation on the theme of kinetic energy. The atoms comprising an object or a gas, are moving around vigorously. In a gas they move freely and bounce off one another. In a solid they vibrate within the structure of the material.

Supplying heat energy to an object causes the atoms to move more rapidly. As a result the temperature of the object increases.

Temperature is a measure of the average kinetic energy of each particle in a mass. For our purposes, it is sufficient to say that heat is a measure of the total amount of thermal energy contained in an object. Our bodies sense temperature but not heat.

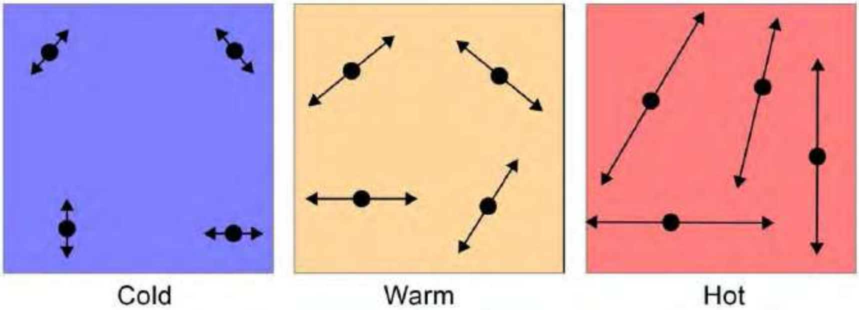


Figure 5.1 Temperature shown as vibration

The Pacific Ocean has a lot more heat than your cup of coffee. But a "hot" cup of coffee will have a higher temperature.

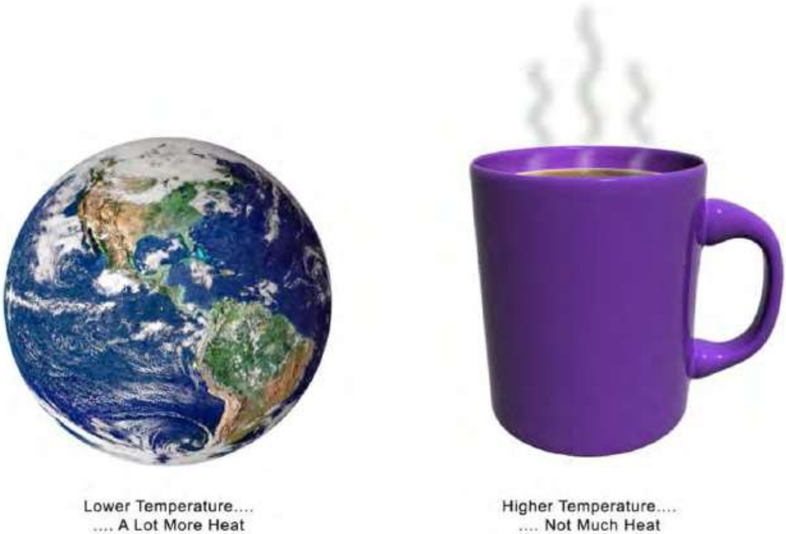


Figure 5.2 The Pacific ocean has a lot more heat that your :"piping hot" cup of coffee

5.3.2 Temperature Scales

It is often convenient to measure temperature on the Kelvin scale. This has a 1:1 equivalence with centigrade but uses a different starting point. Zero Kelvin is the lowest possible temperature in the universe: the point at which no atoms are vibrating. This is equivalent to -273°C.

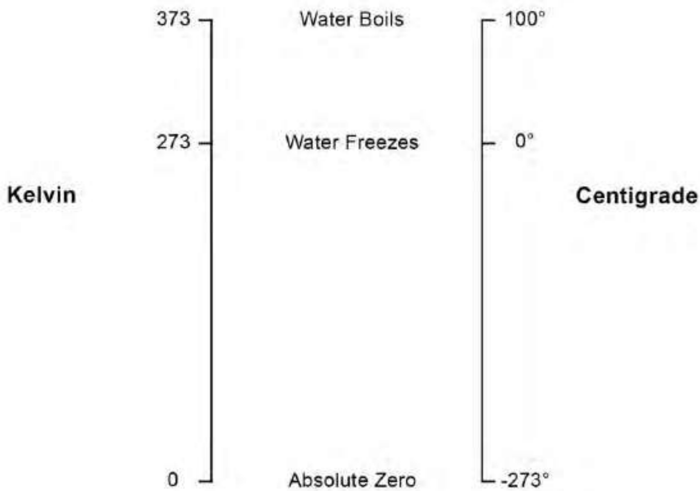


Figure 5.3 Measurements of Temperature

- To convert from Kelvin to °Centigrade subtract 273
- To convert from °Centigrade to Kelvin add 273

5.3.3 Transfer of Heat Energy

When two objects of different temperatures are placed near to each other the higher temperature object will transfer some of its vibrations to the other object. As a result the lower temperature object warms up whilst the higher temperature object cools down.

Section 4

Gravitational Energy

5.4.1 Gravitational Potential Energy

All objects with mass which are in a gravitational field contain a certain amount of stored energy. The amount depends on the object's position in the gravity field. On Earth, when you move an object upwards, it moves in the opposite direction from the gravitational force, so you have to do work to lift it up. The work done adds stored energy to the object.

This is known as *gravitational potential energy* (or simply *potential energy*). Gravitational potential energy (GPE) depends on three things.

- The amount of mass involved.
- The position of the object in the gravity field.
- How strong the gravity is.

The third value is fixed as long as we are considering the Earth, but it would be different on other planets or moons.

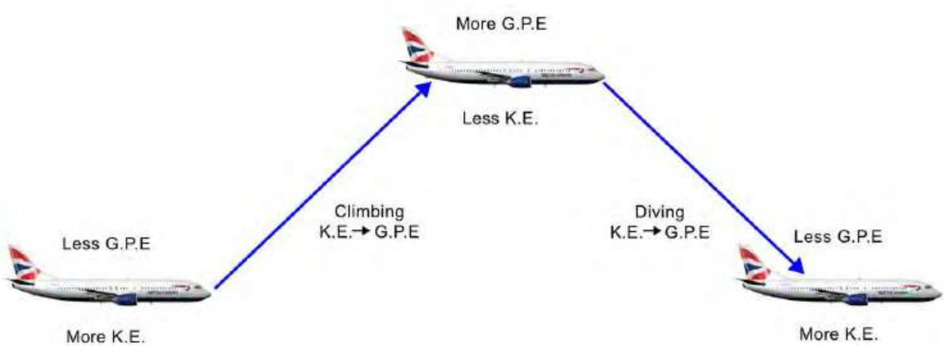


Figure 5.4 Trading kinetic energy and gravitational potential energy

The formula for GPE is written as follows:

GPE = MGH

Where:

GPE = Gravitational Potential Energy (Joules)

M = mass(kg)

G = Gravitational field strength (N/kg) (9.82 for Earth)

H = Height change (metres)

When you let go of an object and let it fall, the gravitational force does work on the object to pull it back down.

Other Energy Sources

5.5.1 Other Energy Sources

Chemical Energy
Energy is stored in the chemical bonds between atoms, we can release it to do work. For example from food via our muscles, or from fuel via an engine.

Electromagnetic Energy
Energy can be stored in the vibrations of electric and magnetic fields, travelling as waves, e.g. light or radio waves.

Spring Energy
If a material is tough enough to cope with being stretched without becoming permanently deformed then the material can 'spring' back to its original shape. We can make a spring from a coil of a suitable material. To compress the spring we have to do some work on it. We can then release the spring to release the stored energy to do work when we need it.

Pressure Energy
A volume of gas can be compressed by doing work on it. We can allow the gas to expand again and do some work when we need it. This is really very similar to compressing a spring.

Sound Energy
Sounds are simply changes in air pressure, so if you can hear something then there is an energy change taking place between the vibration of air molecules (which cause a vibration of your ear drum) to the electrical energy in your auditory nerve.

Section 6

Doing Work

5.6.1 Doing Work

Conversions between forms of energy require work to be done. Let's look at the work done by a force which moves an object.

The work done is calculated by the size of the force x the distance that the object is moved in the direction of application of the force.

Work (Joules) = Force (Newtons) x Distance (metres)

This should seem reasonable, if you push one light and one heavy trolley a certain distance along the harder push required for the heavy trolley means that you have to do more work.

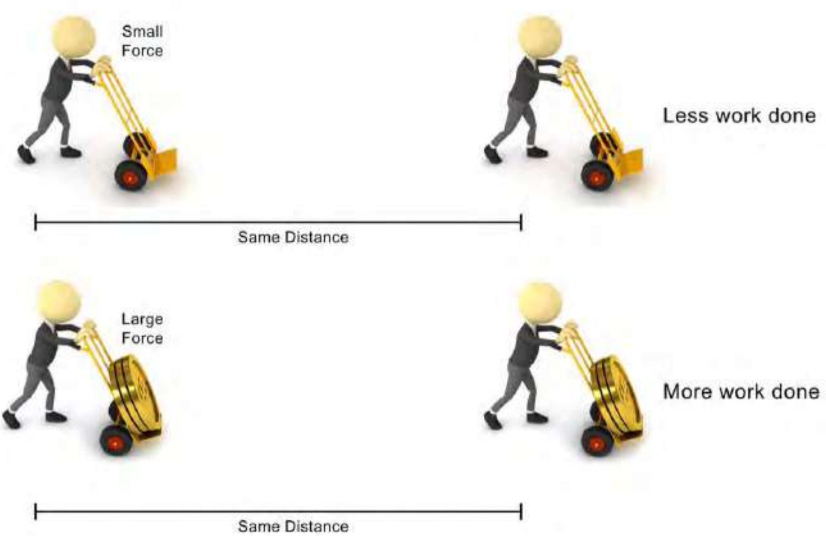


Figure 5.5 Doing work on objects of different mass

If you push two trolleys of the same mass but the second trolley is pushed further this will again require more work.

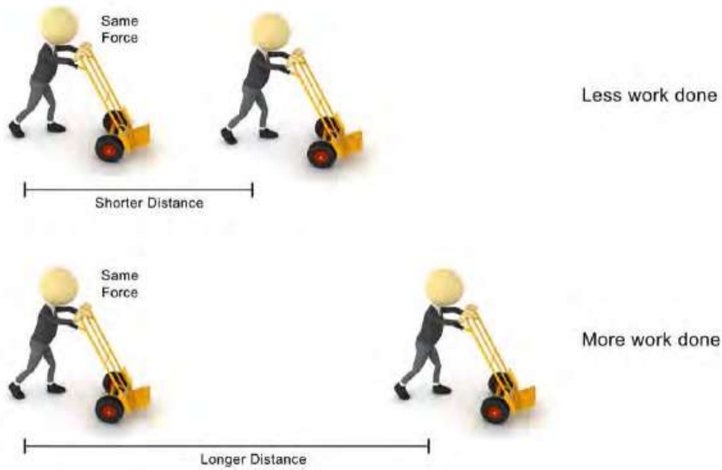


Figure 5.6 Doing work on objects of the same mass

When you first start to push the trolley, you give it kinetic energy. After a short while though, frictional effects will build up and you'll reach a steady speed. At this point the work you are putting in is all lost as heat in the wheel bearings and other sources of friction.

The gravitational potential energy formula we saw earlier is actually an application of this idea.

G.P.E. = MGH

The MG is the object's weight (a force) and the vertical distance moved is the height, so this really is just force x distance. The energy change to move an object up or down is the work needed; when you lift an object up you are doing MGH joules of work on it.

5.6.2 Levers and Moments of Force

The idea that Work = Force x Distance is also quite useful for explaining levers.

A certain amount of work can be done with a small force over a large distance which is then mechanically transferred to become a large force acting through a small distance.

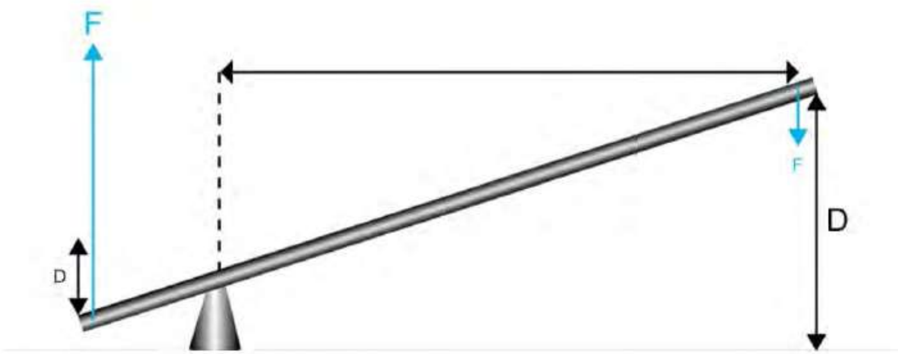


Figure 5.7 The action of a lever

The distance between the point of force application and the fulcrum is known as the *lever arm*.

The amount of work produced by a lever equals the force applied multiplied by the lever arm.

For example a force of three Newtons applied at Point D two meters from the fulcrum would exert the same "moment of force" as one Newton applied six meters from the fulcrum.

5.6.3 Torque

We can further develop the lever arm concept. Instead of a beam rotating around a fulcrum we can have a beam attached to a shaft which is free to rotate. When a force is applied to the end of the lever arm work is done on the shaft: in this case causing it to twist.

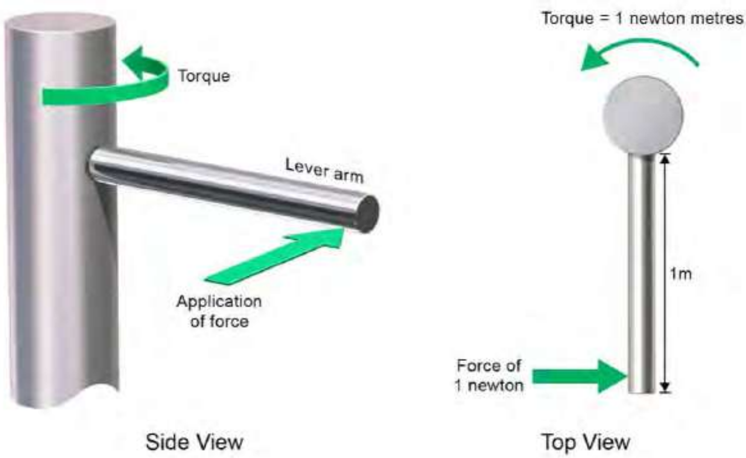


Figure 5.8 Torque force acting on a shaft

In the same way that a moment of force can be used to describe the bending force on a beam, torque is most often used to describe the tendency of a force to rotate an object around an axis. Just as forces are normally thought of as a "push" or a "pull", torque is best described as a "twist".

The amount of torque depends on three quantities:

- The force applied
- The length of the lever arm connecting the axis to the point where the force is applied; and
- The angle between the two

The SI unit of torque is the *Newton Metre*. The Imperial unit is the foot lb.

Moment of force, and *torque* are in fact exactly the same thing it's just that in we tend to use the term torque to describe the twisting action on a shaft and moment of force to describe the action of a lever.

5.6.4 Couples

Let's now look at what happens if we extend the beam right through the shaft. If we now apply forces at both ends of the beam you can see that the shaft will again twist.

Figure 5.9 The action of a couple

When forces act together like this to produce a moment they are said to be *coupled*.

A *couple* is defined as a system of forces which produce a moment.

In an aircraft couples frequently form between various forces. But, unlike our examples above there is no long thin beam and no physical fulcrum. Instead the fuselage itself forms the connecting beam between the point of force application and the axis of rotation. The axis of rotation is often the centre of gravity of the aircraft.

Figure 5.10 Typical force couples acting on an aircraft

Section 7

Energy and Power

5.7.1 Energy and Time

To do anything requires a certain amount of energy. We've seen some formulae, so we could work out the number of Joules of energy required to do certain things, for example accelerating a mass to a particular velocity, or lifting a mass up a certain height.

It's possible to make all sorts of energy calculations, for example, finding how much energy is required to boil a certain amount of water in a kettle, or to mow your lawn, or to drill a hole.

Often though, we are not really worried about how much energy a job requires, but rather how long a job will take. Can you boil a kettle in 30 seconds, or will it take an hour? Or, how long will your car take to accelerate from 0 to 60?

We need some way of relating energy use to time. This quantity is known as Power.

5.7.2 Power

Power is the rate at which energy is being transformed, so it is the energy used divided by the time taken.

Power = Energy/ Time

In S.I. units Power (P) is measured in watts (W). Our energy and time units are the Joule and the Second respectively.

$$\text{Power (Watts)} = \frac{\text{Energy (Joules)}}{\text{Time (seconds)}}$$

Where: 1 watt is one joule per second.

The higher the power, the quicker a given task can be completed. A more powerful kettle will boil water more rapidly.

The concept of power applies to all types of energy transformations. For our studies we are particularly interested in the delivery of mechanical power when using a force to move an object.

We've already seen that the work done by a force is equal to the force multiplied by the distance moved in the direction of application of the force, i.e. the energy transferred by that force.

Energy = Force x Distance

If we consider this per unit of time by dividing both side by time:

$$\frac{\text{Energy}}{\text{Time}} = \frac{\text{Force x Distance}}{\text{Time}}$$

and rearrange for our convenience:

$$\text{Energy} = \frac{\text{Force}}{\text{Time}} \text{ x } \frac{\text{Distance}}{\text{Time}}$$

We can see that: Power = Force x Speed

For example, if a 1000 Newton force is pushing something at a steady 40 metres per second, that's a power of 40 000 watts, or 40 Kilowatts. Each second the applied force is transferring 40 000 Joules of energy to the object. These figures are roughly equivalent to a large family car.

Engine power used to be measured in "horsepower" but in all the best modern textbooks this has been properly superseded by measurement in watts or kilowatts. In some circles (notably car magazines) horsepower is still a popular term, but you should never use it for calculation. 1 horsepower is very approximately 740 watts.

Having talked a lot about energy we now need to consider where all that energy is going to.

Energy Transfer

5.8.1 Energy Transfer

Consider a motionless volume of air before a vehicle travels through it. After the vehicle passes through it, the air is disturbed. It's now moving around, in other words it has gained kinetic energy. The energy gradually spreads itself out and eventually manifests itself as a slight gain in temperature. The energy gained by the air must have come from somewhere. This ties into something we mentioned earlier, namely that moving objects are subject to frictional forces. These forces tend to slow the object down, i.e. make it lose kinetic energy; this in turn explains why friction causes things to heat up. Frictional forces slow things down by converting their kinetic energy to heat.

To stop the object slowing down under frictional forces, we have to keep pushing it. If we provide propulsive energy at the same rate that it is being lost to friction, then we will lose no kinetic energy and the object will maintain the same speed. Consider a racing car travelling along at a constant velocity. Let's assume that its engine is developing 600 kilowatts (kW). Because it is moving at constant velocity the car must be both gaining and losing 600 kW for an overall power flow of zero.

Figure 5.11 Balanced energy in and energy out = constant speed

If the driver lifts his foot off the accelerator, reducing engine power output to zero, then the car will gradually come to a halt as kinetic energy is lost but not replaced (Figure 5.11).

Figure 5.12 Slowing after reducing engine power output to zero

If we increase frictional forces by applying the brakes then the kinetic energy is rapidly converted to heat in the brakes themselves.
This is a classic example of Work = Force x Distance. The bigger the force, the shorter the braking distance required to dump a particular amount of kinetic energy.

Figure 5.13 When using the brakes to slow down the car's kinetic energy is converted to heat energy in the brakes

If we increase the engine power to 700 kW, there will be an initial 100 kW of surplus power that is not lost to friction; this will appear as extra kinetic energy. The result is an acceleration.

Figure 5.15 An excess of power over friction will lead to acceleration

As speed increases power lost increases for two reasons (remember that Power = Force x Speed).

So if the car goes faster, even with the same friction forces, the power loss increases. However, the force itself increases as the car goes faster as well, so the power loss increases quite rapidly and our 'spare' power will reduce from 100 kW to, in our example, 650 kW.

Power-in is again balanced (and speed stabilised) when the 650 kW power-in is balanced by 650 kW of frictional forces.

Figure 5.14 Energy out and in rebalanced - zero acceleration

5.8.2 Vertical Changes

So far we've only considered a vehicle on a level surface. Of course roads, like aeroplanes, often go up or down which means that changes in gravitational potential energy must also be considered.

A car driving up a given slope, at a given speed, will gain gravitational potential energy at a given power. This can also be calculated with the power formula:

Power = Force x Speed

Remember, the speed in question is the speed in the direction of application of the force, and in this case the force is the weight. So it doesn't matter how fast the car is going along, only how fast it is going up.

For example, let's say the car weighs 10 000 Newtons, and is climbing at 0.2 metres per second.

Power = Force x Speed

Power = 10000 x 0.2
= 2 000 watts (2 Kilowatts)

So, compared with driving on the flat, we need to increase engine power in order to maintain the same speed. If we don't, there will be a power deficit and so kinetic energy must decrease at the same rate.

This is consistent with our own practical experience. When you come to a hill if you don't increase power, you'll slow down. If the hill is really steep you may not have enough engine power to maintain speed even at full throttle.

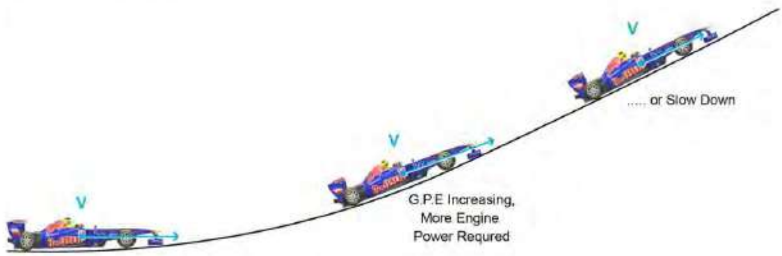


Figure 5.16 Climb requires more power

The converse happens when going downhill. If you don't ease off the accelerator then you'll speed up. If the hill is steep enough the car will speed up even with no engine power applied. In this case you must either accept the speed increase or apply the brakes to convert more power to heat.

Figure 5.17 Descent requires less power

In an aircraft we can easily trade between kinetic and potential energy by climbing or descending.

5.8.3 Section Summary

In summary, we can see that operating a vehicle involves multiple forms of energy, all being changed from one form to another. This leads to another important observation; although Total Energy is conservative, individual forms of energy are not.

CONCEPTS		UNITS	
NAME	SYMBOL	NAME	SYMBOL
Energy	E	Joules	J
The ability to do work. Never created or destroyed, but changes form. All forms are scalars. K.E. = $1/2\ MV^2$ G.P.E. = MGH Heat is Thermal Energy. Work done by a Force = Force x Distance			
Temperature	T	Kelvin	K
Kelvin = Centigrade +273 Heat and temperature do not mean the same thing			
Power	P	watts	w
Rate of doing work, Joules per second. Power developed by a Force = Force x Speed			

Figure 5.18 Summary of Units

Section 9

Momentum and Kinetic Energy

5.9.1 Momentum and Kinetic Energy

Momentum is not the same as kinetic energy. Look at the formulae for each:

Momentum = MV so K.E. = 1/2 MV^2

Because both formulae include mass and velocity, it's easy to see where the idea comes from that the two things are similar; but remember:

Momentum:

- Is a vector quantity
- It is always conserved in any interaction between multiple objects; if you know the total momentum of all the objects involved then this will remain unchanged, if one object changes momentum then another object must also change by an equal and opposite amount.
- We can use momentum based calculations to see how much time it takes for a force to cause a certain change in velocity.

Kinetic energy:

- Is a scalar quantity
- It is not conservative; the total kinetic energies of the objects in a system may change as energy is converted to or from other forms.
- We can use kinetic energy based calculations to see how much distance it takes for a force to cause a certain change in velocity.

Section 10

Practical Examples

5.10.1 Consolidation

It's time to consolidate what we have learned so far. We'll do this with some real-world examples to help you relate theory to practice.

Consider the space station orbiting the Earth. We'll assume that the orbit is perfectly circular, and that it's high enough to be clear of the atmosphere so we can forget about air resistance.

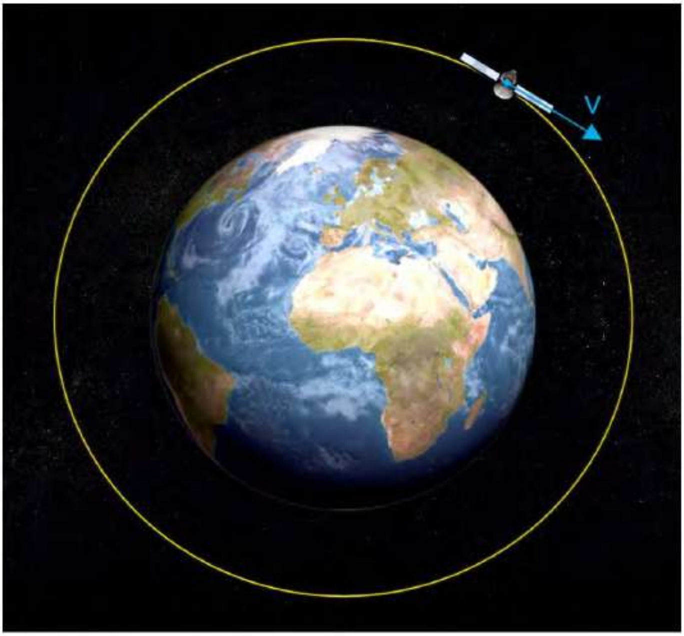


Figure 5.19 The space station in orbit

When we see images of the astronauts floating inside the space station it is easy to assume (wrongly) that things in orbit are weightless.

Figure 5.20 The illusion of weightlessness in orbit Image courtesy of NASA

We know that an object will only move along a circular path if it is subject to acceleration inwards towards the centre of the circle. For the path to be perfectly circular, the force causing the acceleration needs to be at right angles to the direction of motion. For orbital motion, the force responsible for this is weight.

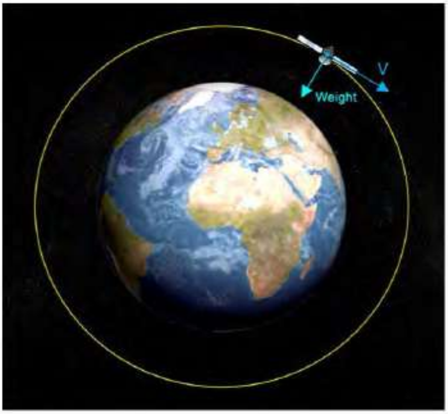


Figure 5.21 Weight force keeps the space station in orbit

If there was no weight, the space station would move in a straight line, in accordance with Newton's first law and would disappear off into the depths of the Solar System. Objects do not stay in orbit because they are weightless, they stay in orbit because they have weight!

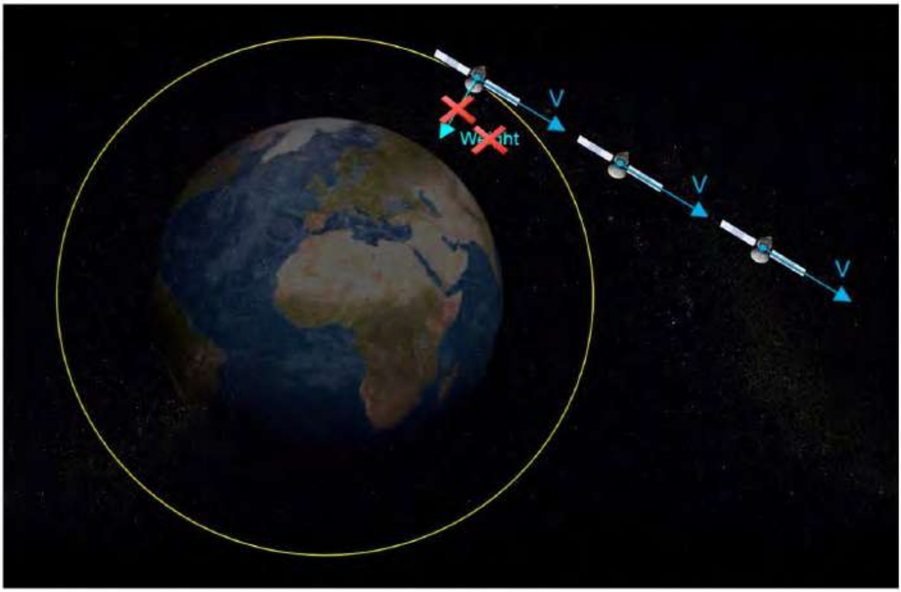


Figure 5.22 Without weight the space station would leave orbit

To understand what's happening, it's worth thinking about how we perceive weight.

You can't actually perceive the force of gravity acting upon you, because it acts evenly on every molecule in your body. Imagine floating inside a windowless box, you would have no way of knowing if you were subject to a gravitational effect.

You could be floating in deep space without accelerating or you could be in orbit around the Earth, with an acceleration of about 10 m/s^2. But equally you might be in orbit around Jupiter, now with an acceleration of 25 m/s^2. You could even be falling towards a distant black hole with an acceleration of 1000 m/s^2. In all these cases all parts of your body would have the same acceleration so you would feel nothing.

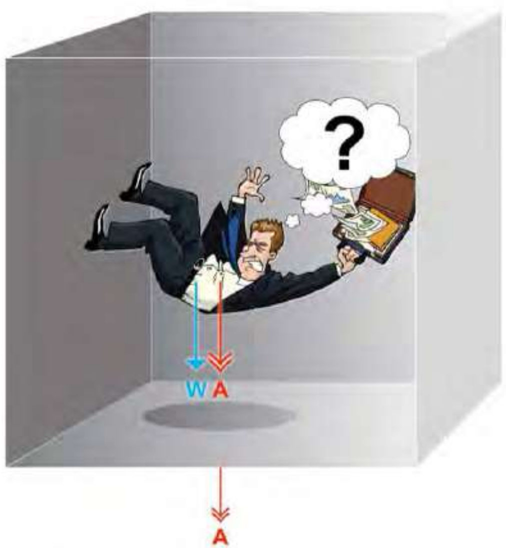


Figure 5.23 In orbit or in deep space?

If we are in a gravitational field and we are not falling there must be a force equal and opposite to our weight acting upwards. We can feel this force because it is applied unevenly to our body, for example through our feet when we are standing. The force is then spread to the rest of our body, causing body tissue to become squashed.

It's this squashing of our bodies that allows us to perceive our weight.

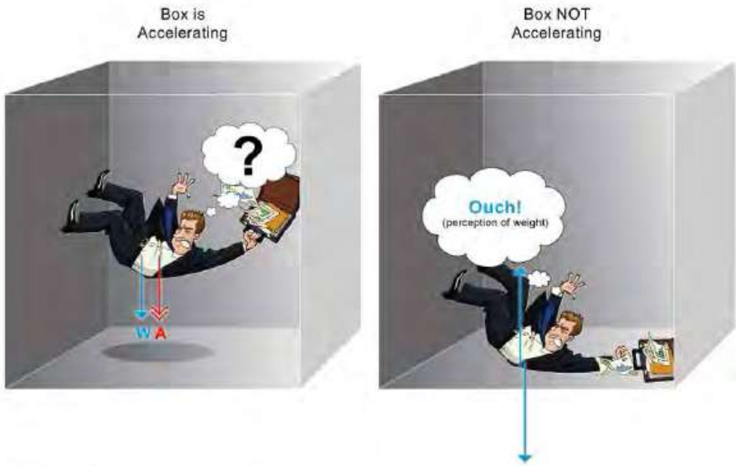


Figure 5.24 Perceiving weight

If you now imagine being inside a high performance aircraft that is manoeuvring hard, you will feel as if your weight has increased, but in fact your weight is the same as it always is. If your mass is 70 kilograms, you will weigh 700 Newtons. And you will still weigh 700 Newtons whatever your aircraft is doing.

What is actually happening is the aircraft is accelerating you in the direction of the manoeuvre. To do this it must apply a force to you which is transmitted through your body. You perceive the increased squashing effect as an increase in weight.

5.10.2 Momentum and Energy

Now that we've established the idea that an object in orbit is still subject to weight, let's look at the motion in terms of momentum and energy.

When an object is in orbit it's speed is constant but its direction is constantly changing. Hence the vector quantities of velocity and momentum are both changing continually. If the momentum is changing, then in accordance with Newton's second law (the rate of change of momentum of an object is equal to the applied force) there must be an applied force. This is consistent with the idea that weight is still present.

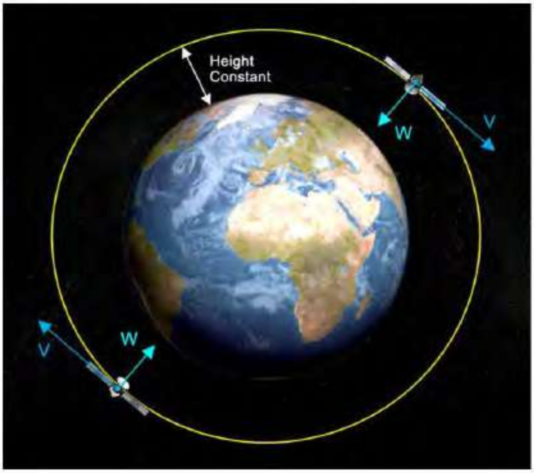


Figure 5.25 Orbit - constant speed and distance - constant GPE

However, if the speed of the object is constant then the kinetic energy is not changing either. This might sound a bit strange; how can a force be acting but not doing any work?

To do work the force must move the object in the direction of application of the force. Yet with a circular orbit the force is always at right angles to the direction of motion, so no work is done.

We can also look at this situation from a Gravitational Potential Energy point of view; as the orbit is circular the object is orbiting at a constant height and so has constant potential energy. This is consistent with the idea that gravity is not doing any work on the object.

6

Gyroscopes

A gyroscope comprises a rotating mass mounted on a shaft which allows it to spin. The shaft itself is mounted on at least two "gimbals". The gimbals allow the rotor axis to take up, or be set at, any attitude.

When physicists first created the device they noticed that it possessed two very unusual characteristics: rigidity and precession

Section 1

Rigidity and Precession

6.1.1 Introduction

The final task in our study of mechanics is to understand the properties of gyroscopes.

In the conventional sense a gyroscope comprises a rotating mass (the rotor) mounted on a shaft which allows it to spin. The shaft itself is mounted on at least two "gimbals". The gimbals allow the rotor axis to take up, or be set at, any attitude. The figure below shows the arrangement.

Figure 6.1 A typical gyroscope arrangement

When physicists first created the device they noticed that it possessed two very unusual characteristics which later came to be known as *rigidity* and *precession*.

6.1.2 Rigidity

As the spin speed of the rotor is increased the gyroscope's attitude becomes more and more rigidly fixed. For example, if you set the spinning gyro so that its axis of spin points vertically upwards it will remain fixed in this position until an external force is applied to the rotor. But the really important point is that the gyroscope's attitude is not fixed relative to the Earth, it is fixed in space. For example, if you were to move the gyroscope around the surface of the Earth, it would appear to change its attitude. But in fact its attitude hasn't changed at all - what has changed is the reference plane of the Earth! Figure 6.2 shows what's happening.

The amount of rigidity possessed by a gyroscope depends on three factors.

- The amount of mass in the rotor
- The distance of the mass from the axis of spin
- The speed of spin

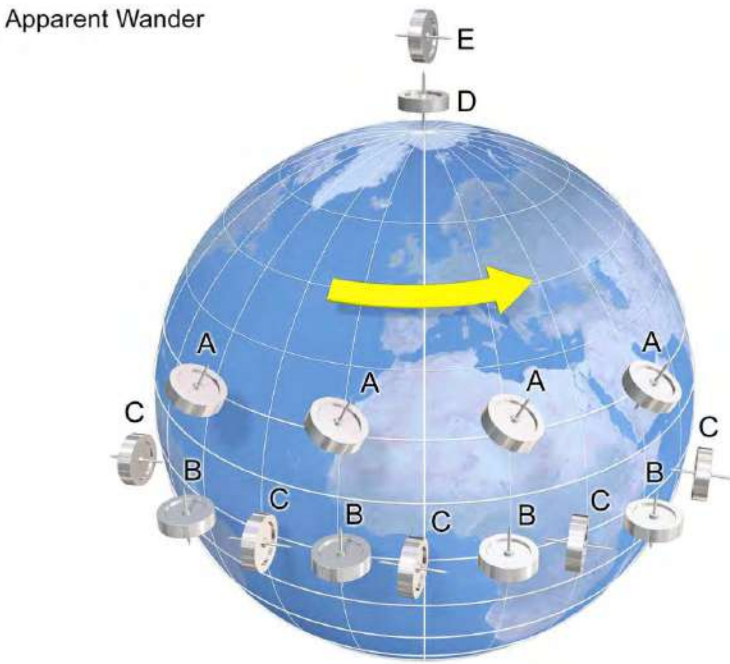


Figure 6.2 Gyroscopic rigidity in practice

6.1.3 Precession

The other peculiar characteristic of gyroscopes is precession. If you apply an external force to a spinning rotor the rotor will respond as if the force has been applied 90° further on in the direction of rotor rotation.

For example, applying a sideways force at the bottom of a stationary rotor would cause it to tilt towards the left. But if you apply the same force to the spinning rotor shown it will actually swivel rather than tilt.

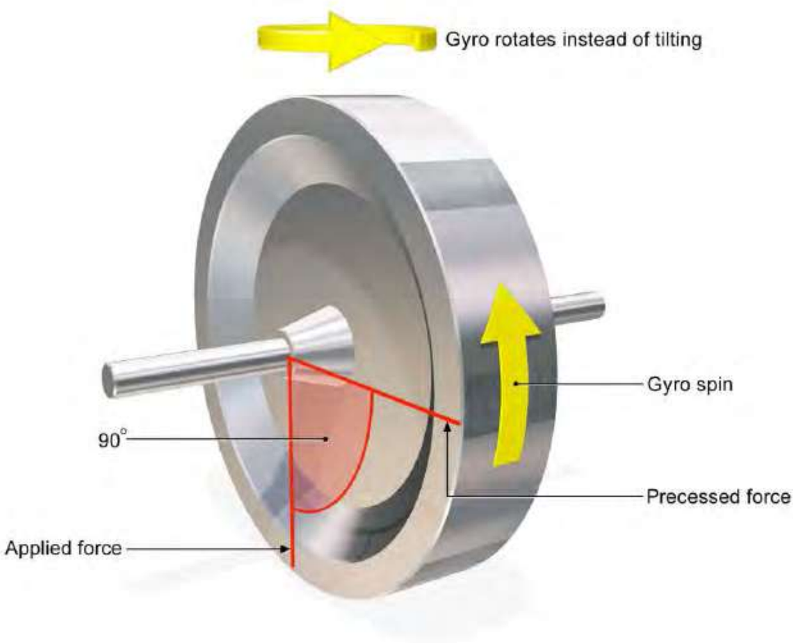


Figure 6.3 Gyroscopic precession in practice

Section 2

Gyroscopic Behaviour

6.2.1 Gyroscopes and Aircraft

You might be wondering what all this has to do with aircraft! The point is that any object with mass can possess gyroscopic properties - provided it is spinning. It doesn't have to be mounted in gimbals as per the classic gyroscopic apparatus - all it needs is to be spinning (usually around its own centre of gravity), and more or less free to move.

It should take no leap of imagination to realise that a spinning aircraft will exhibit the same properties of rigidity and precession. Both factors must be taken into account whenever we wish to study the effect of a control or aerodynamic force when it is applied to a spinning aircraft.

Gyroscopes are also found in instruments, notably the attitude indicator and the turn indicator. Understanding the fundamental properties of gyroscopes will help you later in understanding the errors which occur when they go wrong or lose power.

7

The Physics of Gases

A gas is a state of matter comprising molecules, atoms, electrons and ions which are not bound together and which are in a more or less constant state of random motion.

Section 1

Gas Properties

7.1.1 The Nature of Gases

Because it comprises matter, gas has mass. But, unlike solid matter, a quantity of unconstrained gas will have no definite shape or volume.

Compared to solid and liquid forms of matter, gases have relatively low density and low viscosity. They also expand and contract greatly with changes in pressure or temperature and are thus considered to be very compressible.

Gases also diffuse easily. Any gas placed in a container will rapidly spread within the container until it is evenly distributed throughout it's volume.

7.1.2 Properties of a Gas

Gases posses the same properties of matter as solids and liquids.

Temperature

Temperature is the measure of the average kinetic energy stored in a particle. So the moving particles in a gas are a measure of its temperature. The faster the particles move the greater the temperature.

Pressure

The particles comprising a gas are constantly moving. When they collide with a surface they exert a force on it which is felt as pressure.

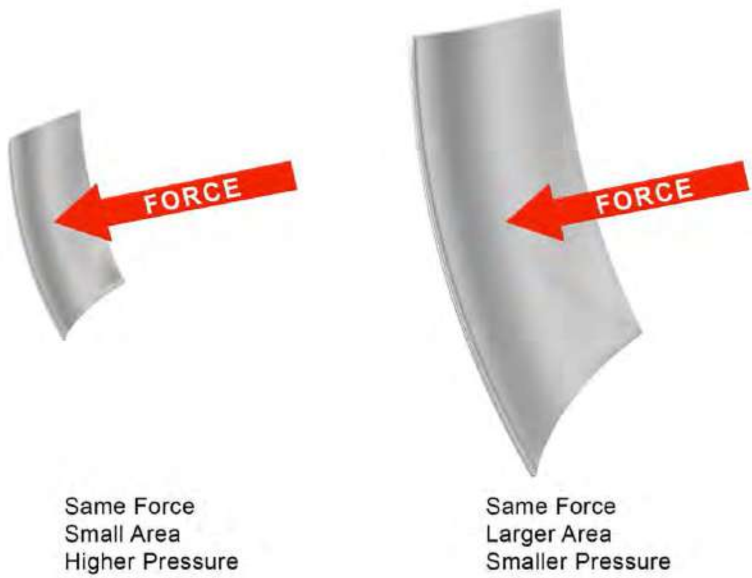


Figure 7.1 Pressure force and area

Pressure (P) is the force developed over a unit of area. The SI unit for pressure is the Pascal (Pa), where 1 Pascal is one Newton per square metre.

Pressure = Force / Area

Density

The particles comprising a gas are free to move about so the most useful measure of gas is its density. Density is the mass of a substance per unit volume. Its symbol is the Greek letter rho (ρ).

There is no unique name for its units, we just refer to it as kilograms per cubic metre.

Density (ρ) = Mass / Volume

For a fixed mass of gas, *density is inversely proportional to volume.*

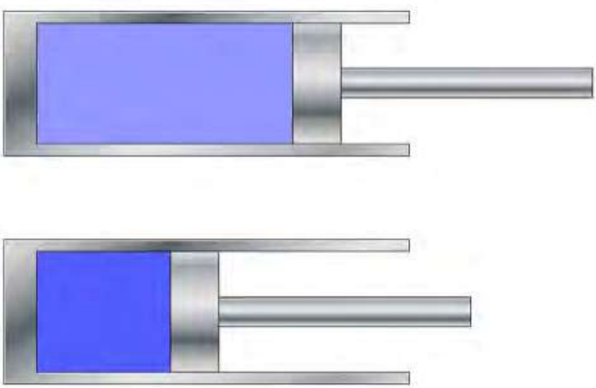


Figure 7.2 A fixed mass of gas in two different volumes results in a change in density

Section 2

The Gas Laws

7.2.1 The Gas Laws

Physicists have discovered that there are a number of predictable relationships that hold true for all gases. These are known as the Gas Laws. There are some limits to these laws, for example if a gas is cooled down enough, it will turn into a liquid. This will happen at very different temperatures depending on the gas. But as long as a substance is in its gas state then it will behave almost identically to all the other gases.

Charles' Law

For Charles Law Constant Pressure French physicist Jacques Charles discovered a relationship between the temperature of a gas and the volume it occupies.

Let's assume we have a fixed mass of gas, and we keep its pressure constant; the easiest way to imagine that is with a sealed piston as shown below.

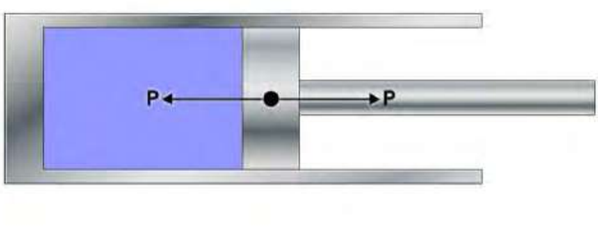


Figure 7.3 Charles' Law Apparatus

The piston forms a gas tight seal, so we know we have a fixed amount. It is also free to move backwards and forwards so the contents will be at atmospheric pressure; if anything happens to increase pressure the piston will expand, and vice-versa.

Charles discovered that in these conditions, the volume of the gas is directly proportional to the temperature in Kelvin.

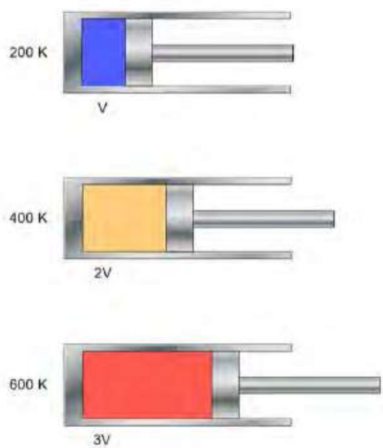


Figure 7.4 Charles' Law - volume proportional to temperature

So, if we change the temperature of the gas and allow it to change volume, we'll see that doubling the temperature doubles the volume and so on.

For a constant mass of gas, at a constant pressure:

$V \propto T$

Graphically this is a straight line that would, if extended, pass though zero volume at zero temperature. However the gas will condense before then, so the actual behaviour doesn't extend that low.

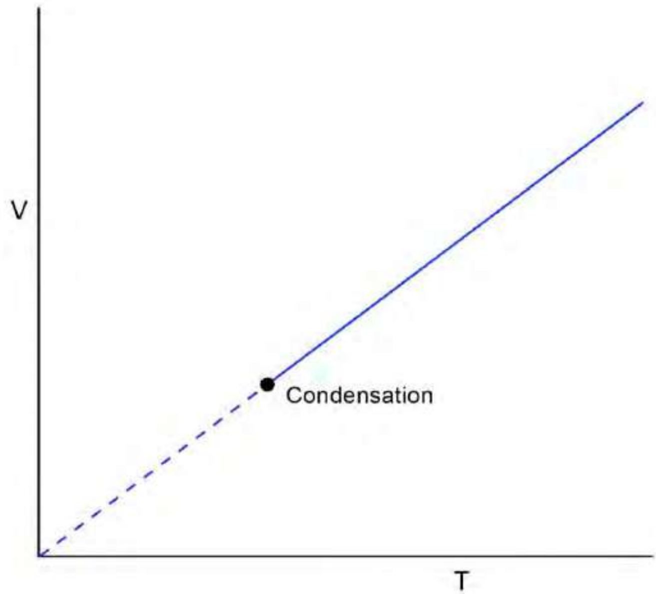


Figure 7.5 Charles' Law

Boyle's Law

Robert Boyle discovered a relationship between the volume of a gas and its pressure. Again, imagine we have a cylinder. This time though, we'll pull or push on the piston to change the volume of the gas and see what happens to its pressure.

Boyle discovered that pressure and volume are inversely related, so when the volume is decreased (by pushing the piston inwards) the pressure increases.

To properly explore the pressure and volume relationship, we need to maintain a constant temperature. Unfortunately the very act of compressing or expanding the gas will also make it change temperature. So let's imagine we have some sort of device that will hold the temperature constant.

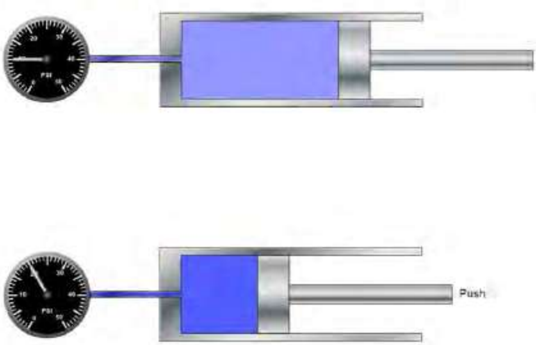


Figure 7.6 Boyle's Law apparatus

Mathematically this can be expressed as $P \propto \frac{1}{V}$

Gay-Lussac's Law (the Pressure Law)

Joseph Gay-Lussac discovered a relationship between the pressure of a gas and its temperature. This is demonstrated by having a container with a pressure gauge. The container has a constant volume and is sealed to contain a fixed mass of gas.

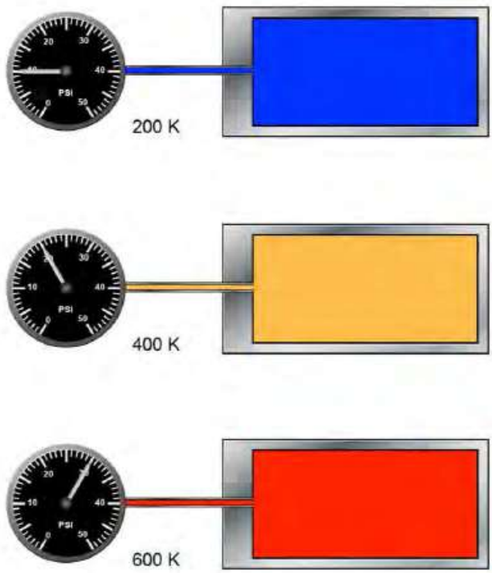


Figure 7.7 Gay-Lussac's Gas law

By varying the temperature of the gas the effect on the pressure can be observed. Gay-Lussac discovered that the pressure of the gas was directly proportional to the absolute temperature.

For a constant mass of gas at a constant volume:

$P \propto T$

7.2.2 Combining The Gas Laws

So far we have seen 3 individual gas laws:

Charles' Law	$V \propto T$	(constant pressure)
Boyle's Law	$P \propto 1/V$	(constant temperature)
Gay-Lussac's Law	$P \propto T$	(constant volume)

These can be combined to produce what is known as the *universal gas law*. Getting to the universal law requires moderately complex algebra which you don't need to know. If you are working with a fixed mass of gas then it turns out the following expression is true:

$$\frac{PV}{T} = \text{Constant}$$

This tells you every relationship you need to know:

- If you fix the pressure, then if V is increased T must also have been increased to keep the constant fixed. This shows us that if P is fixed V is proportional to T, which is Charles's law.
- If you fix the temperature then if P goes up V must go down, and vice versa. In other words they are inversely proportional. This is Boyle's law.
- If you fix the volume, then if P is increased T must also have increased. P is proportional to T, which is Gay-Lussac's law.

Section 3
The Atmosphere

7.3.1 The Atmosphere

The Earth's atmosphere is a mixture of gases. Approximately 78% of the gas comprising the atmosphere is nitrogen. The next most predominant gas is oxygen at 21%. The remaining 1% comprises a mixture of other gases. The common name for this particular mixture of gases is *air*.

The Earth's atmosphere extends from the surface upwards to a height of about 100 km. The weight of a column of air 100 km high is very significant and so a considerable amount of compression of the air occurs lower down in the column. Consequently, air pressure and density are generally highest at the surface of the Earth and decrease with height.

From the surface to approximately 11 km temperature also decreases with height as we might expect from our knowledge of the gas laws. Between about 11 km and about 20 km the temperature remains constant. Thereafter it increases with height, thanks to the increasing influence of solar radiation.

7.3.2 The International Standard Atmosphere

Air pressure, density and temperature are by no means constant throughout any given horizontal layer of the Earth's atmosphere. Numerous influences cause local variations so, for the purposes of aviation, an International Standard Atmosphere (ISA) has been defined. The ISA makes the following assumptions:

- Sea level Temperature: 15° C.
- Sea level Pressure: 1013.25 hectopascals
- Sea level Density: 1.225 kg/m^3

A hectopascal is 100 pascals, the SI unit of pressure. 1 hectopascal is very approximately 0.1% of atmospheric pressure at sea level.

ISA also assumes a standard reduction of temperature with altitude: a decrease of 1.98°C per 1000 ft up to a height of 11 km. Thereafter the temperature is assumed to remain constant at -56.5°C.

Although you will rarely, if ever, encounter actual ISA conditions you will frequently use the known deviation from ISA, to make planning and performance calculations. ISA can be represented graphically as shown below.

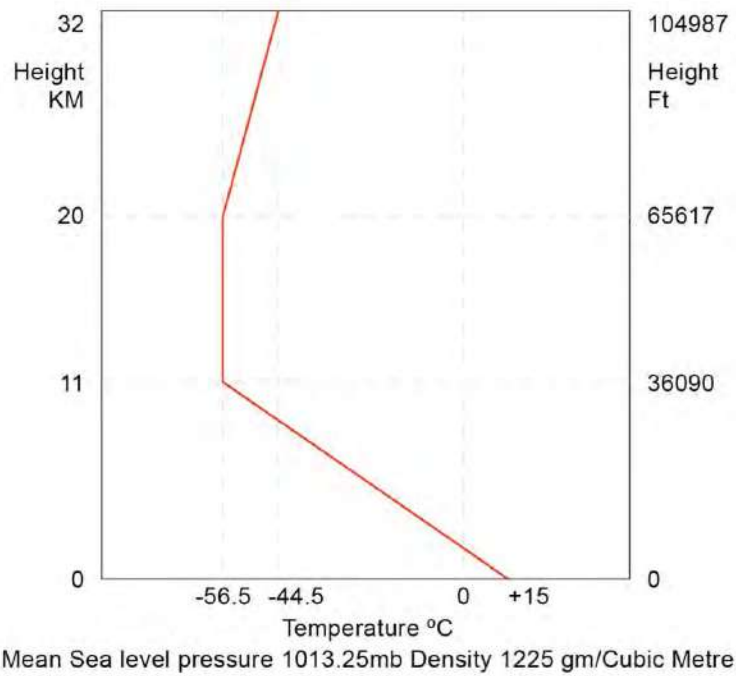


Figure 7.8 The International Standard Atmosphere

7.3.3 Humidity

Humidity is the term used to describe the amount of water vapour in a gas. A molecule of water is less massive than either a molecule of nitrogen or of oxygen.

For any gas, at a given temperature and pressure, the number of molecules present is constant for a particular volume. So, for a given volume of dry air, when a number of water molecules are added, the number of nitrogen or oxygen atoms must reduce by the same amount.

This means that the mass per unit volume (density) of humid air is less than the density of dry air. Density decreases with increased humidity.